NBER WORKING PAPER SERIES

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Working Paper 34896
http://www.nber.org/papers/w34896

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
February 2026

We thank Boston University, Babson College, and the University of Piraeus for research support. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

At least one co-author has disclosed additional relationships of potential relevance for this research. Further information is available online at http://www.nber.org/papers/w34896

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JEL No. E0, E39

ABSTRACT

This paper addresses two fundamental macroeconomics questions. First, are macro shocks large enough to alter the course of the economy? Second, are they large enough to materially impact economic welfare?Lucas and many others have addressed these issues, but do so primarily in the context of representative agent models. We study these questions using a large-scale, general equilibrium, stochastic, overlapping generations model. We consider 80 generations overlapping in an economy buffeted by realistically calibrated total factor productivity and capital depreciation shocks. The model is solved using Marcet's projection method taking explicit account of the full state space, which comprises 81 variables. Our findings, some recapitulated from prior studies by Hasanhodzic and Kotlikoff, suggest macro shocks are second order both with respect to their impact on aggregate variables and individual welfare. Specifically, the probability that the stochastic economy's long-run aggregates materially deviate from their deterministic counterparts is less than one percent. Furthermore, the realized (simulated) lifetime utility of generations born in the long run rarely differs from deterministic long-run utility levels by more than 1 percent, measured as consumption-compensating differentials. These findings are supported by the model's small equity premium. Moreover, they are essentially indifferent to the presence of a bond market. Both results suggest agents are minimally concerned with precautionary savings against these shocks. Our RBC-in-OLG findings suggest that what really moves the macroeconomy and demands attention is policy, not shocks.

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1 Introduction

As any LLM will confirm, shocks and short-run stabilization policy comprise macroeconomists' bread and butter. But are the shocks large enough to matter to long-term macroeconomic aggregates or household well-being? This paper uses Hasanhodzic and Kotlikoff's (2018, 2025b) large-scale, general equilibrium, OLG RBC model to address this question.

Documenting that financial markets overreact to macro fluctuations has a long and distinguished pedigree. Excess volatility, the equity premium puzzle, noise trading, irrational exuberance, bank runs, multiple equilibria, and behavioral finance reference research seeking to explain seemingly excessive financial market responses. Our focus is not on overreactions to significant macro shocks, but on the absence of meaningful macro risk over which to react. If, as argued here, macro shocks are second order in large-scale life-cycle models, which, unlike representative agent models, appear realistic, an important policy message follows. Fiscal policy, particularly intergenerational redistribution policy, is the singular means by which the economy's future can be worsened or improved.

Our model features isoelastic preferences with moderate risk aversion, Cobb–Douglas production, shocks to total factor productivity (TFP), and shocks to the rate of capital depreciation. The log TFP process is trend-stationary with normally distributed innovations, consistent with standard macroeconomic growth models.[1] TFP shocks alone are sufficient to match the variability of most macroeconomic aggregates, such as output and consumption, with the exception of the rate of return on capital. Introducing depreciation shocks improves the model's ability to replicate the variability of returns to national wealth and breaks the otherwise perfect correlation between capital returns and wages. We solve the model both with and without generational policy, specifically pay-go Social Security, which we effect via a fixed-rate payroll tax.[2] Agents live for 80 years, corresponding to ages 20–100. The overlap

[1]See, e.g., Hansen (1985), Prescott (1986), Cooley and Prescott (1995), R'ios-Rull and Santaeulalia-Llopis (2010), Gomme, Rogerson, Rupert, and Wright (2005), and Judd, Maliar, and Maliar (2011).

[2]As shown in Green and Kotlikoff (2008), there is a continuum of ways to describe any fiscal policy. Thus, what we call pay-go Social Security policy could equally well be called deficit policy or, indeed, surplus policy

of any generation with, as they age, a total of 158 other generations permits significant intercohort risk sharing. We facilitate such risk sharing by positing a one-period bond market.

We demonstrate the second-order nature of macro shocks in several ways. The first draws on the generational risk measures developed in Hasanhodzic and Kotlikoff (2018, 2025b). Their method evaluates the extent to which the state of the economy at birth affects a cohort's realized lifetime utility. Specifically, for each cohort born after year 300, they compute the compensating-consumption factor, λ, that must be applied to that cohort's realized consumption path to yield the same realized lifetime utility as the post–300 average. They then take the average absolute deviation of λ from one. The closer this average is to zero, the less one's date of birth matters for lifetime welfare.

Under our baseline calibration, which only assumes TFP shocks, the average absolute generational risk is 1.40 percent, with a standard deviation of 1.29 percent and a maximum value of 4.95 percent. With Social Security removed, the value is very similar at 1.46 percent. These are extremely small numbers. They indicate that the lifetime utility of someone born in a "bad" state of nature differs little from that of someone born in an average or "good" state. They also show that Social Security does not materially share generational risk. One explanation, conveyed in Brumm, Feng, Kotlikoff, and Kubler (2022, 2024), is that pay-go Social Security, which features unidirectional transfers, is not fundamentally designed to share risk across generations.[3] The other is that macro risks are too small to matter whether or not they are shared across generations. When depreciation shocks are added, generational risk increases modestly. Without Social Security, the average absolute adjustment increases to 2.13 percent. This is intuitive. Depreciation shocks disproportionately affect older households, generating an additional source of idiosyncratic variation that does not symmetrically impact the young. When Social Security is included, its risk-sharing properties lower the

coupled with alternative cohort-specific time paths of taxes and transfers.

[3]Bidirectional transfer policies in which younger generations transfer to older generations in states that are favorable to the young and the opposite occurs in states that are favorable to the old would be so designed.

average adjustment to 1.96 percent. But even these values are small.

Next, we show that macroeconomic impacts of policy changes in the stochastic version of our model are, on average, trivially different from those arising in the deterministic version of our model. Across all calibrations, the stochastic model delivers macro responses that are virtually indistinguishable from those generated by the deterministic version. This insight, originally signaled by Ríos-Rull (1994), implies that traditional deterministic OLG models – such as Auerbach and Kotlikoff (1987) – can adequately capture key long-run effects of intergenerational redistribution and other policies.

Third, we show that the economy's long-run aggregates along its stochastic paths are almost always very close to their deterministic counterparts. This underlies point four. Economies that are running apparently unsustainable fiscal policies – policies that will impose major fiscal burdens on future generations – cannot count on good luck to save their day. Yes, AI appears transformational. But will it produce sustained, major TFP shocks that far rival those of the past? Argentina is a case in point in this context. Its per capita GDP equaled 85 percent of the US value in 1925. Today that figure is 14 percent notwithstanding the advent of amazing technological change, including nuclear energy, the transistor, antibiotics, computers, and the internet – all of which has been available to Argentina.

Finally, we recapitulate Hasanhodzic and Kotlikoff's (2018, 2025a, 2025b) finding that the economy's equity premium is tiny unless one makes extreme risk-aversion assumptions. Reasonably risk-averse households aren't willing to pay much to avoid the modest uncertainty they face from investing in capital. Indeed, households are essentially indifferent between holding bonds and holding capital, and the aggregate statistics of the model are nearly identical regardless of whether bonds are available.

We proceed by reviewing the literature, detailing our model, specifying its calibration, describing the solution method, presenting results, and concluding. As described in section 5, we solve our model to high precision using the algorithm of Hasanhodzic and Kotlikoff (2018, 2025b), which in turn builds on Marcet's (1988) projection method and related research by

Judd, Maliar, and Maliar (2011). Unlike Krusell and Smith (1998), our model incorporates the full state space. This encompasses 80 state variables comprising 79 generation-specific levels of cash-on-hand and the lagged TFP shock.

2 Literature Review

Lucas (1987) is a seminal quantitative assessment of macroeconomic fluctuations and their impact on welfare. He shows, in a representative-agent model with CRRA preferences, that the welfare gains from eliminating business-cycle fluctuations are negligible – particularly compared to the impact of small increases in the growth rate of consumption. Obstfeld (1994) and Dolmas (1998) revisit Lucas's welfare-cost calculation, but posit non-expected utility, including Epstein-Zin preferences. They find larger business-cycle welfare costs, but still relatively modest impacts on economic aggregates. Lucas' proposition – that macro risk doesn't matter or doesn't matter much – has since been reaffirmed in various frameworks apart from that considered here – a large-scale, general equilibrium OLG/RBC model. The prior post-Lucas literature includes general-equilibrium settings with non-time-separable preferences (Otrok (2001)), empirical assessments of developing-country data by Pallage and Robe (2003), and studies using risky asset returns to proxy for aggregate consumption risk (e.g., Alvarez and Jermann (2004)). As mentioned, Hasanhodzic and Kotlikoff (2018, 2025b) show that the findings of this literature, which have been primarily based on representative agent or heterogeneous, representative-agent models, apply to OLG/RBC models.

A parallel strand of literature investigates whether smooth aggregate consumption can be reconciled with the high volatility and large equity premiums observed in financial markets. Constantinides and Duffie (1996) demonstrate that under imperfectly insured idiosyncratic income risk, aggregate risk and welfare effects can remain small even while risky asset prices exhibit high volatility. This reconciliation can also be achieved by invoking habit-persistent preferences, as discussed by Campbell and Cochrane (1999). Storesletten, Telmer, and Yaron

(2007) extend this insight by linking, in a partial equilibrium life-cycle model, the negative relationship between idiosyncratic income risk and the economy's performance (bad aggregate states and low equity returns coincide with high income volatility).

Several models also suggest that relatively smooth aggregate consumption can coexist with considerable variability in household-level consumption due to uninsurable idiosyncratic shocks. Krusell and Smith (1998) is a case in point. There is considerable variation in the economic experiences of its heterogeneous representative agents with, nevertheless, rather muted impacts of business-cycle shocks. Heathcote, Storesletten, and Violante (2009) report similar findings – a result supported by survey findings of Blundell, Pistaferri, and Preston (2008). Kaplan and Violante (2010) posit a partial equilibrium life-cycle model, pointing out the significant ability of households to smooth consumption over time, while Guvenen, Ozkan, and Song (2014) argue that the risk of individual earnings, rather than aggregate fluctuations in the business cycle, is the main driver of welfare differences.

Finally, Harenberg and Ludwig (2015) examine the interaction between aggregate and idiosyncratic risk within OLG models. They also show very limited cost, as measured by the equity premium, of aggregate shocks in the context of idiosyncratic risk and household heterogeneity. Indeed, their model suggests that, at least in an OLG context, the interaction between aggregate and idiosyncratic shocks can dampen, rather than exacerbate the impact of macroeconomic risk.

3 The Model

3.1 Endowments and Preferences

The economy is populated by 80 overlapping generations. Each generation lives from age 1 to age G, of 80. It works through retirement age, R, of 45. All agents within a generation are identical and are referenced by their age g at time t. Each cohort supplies ℓ_g units of labor per period, which equals 1 before and 0 after retirement.[4] Utility, in a given year, is time-separable and isoelastic, with risk aversion coefficient γ. Thus,

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}. \tag{1}$$

3.2 Technology

Production is Cobb-Douglas with output, Y_t, given by

$$Y_t = z_t K_t^{\alpha} L_t^{1-\alpha}, \tag{2}$$

where z_t is total factor productivity, α is capital's share, K_t is capital, and L_t is labor demand. Equilibrium factor prices satisfy

$$w_t = z_t(1-\alpha)\left(\frac{K_t}{R}\right)^{\alpha}, \tag{3}$$

$$r_t = z_t\alpha\left(\frac{K_t}{R}\right)^{\alpha-1} - \delta_t, \tag{4}$$

with i.i.d. depreciation shock, $\delta_t \sim \mathcal{N}(0, \psi^2)$, for each t. Total factor productivity, z_t, obeys

$$\ln(z_{t+1}) = \rho \ln(z_t) + \epsilon_{t+1}, \tag{5}$$

where ρ is a positive constant and ϵ_t is an i.i.d. process with $\epsilon_t \sim \mathcal{N}(0, \sigma^2)$, for each t,

3.3 Financial Markets

Households save and invest in risky capital and, when available, one-period safe bonds. In the presence of the bond market, investing 1 unit of consumption in bonds at time t yields $1 + \bar{r}_t$ units of the model's single good in period $t + 1$. The return, $\bar{r}_t$, is indexed by t – the

[4] Hence, the total labor supply, R, is constant and equals 45.

time it's determined. The asset demand of a household age g at time t is given by $a_{g,t}$ and its share of assets invested in bonds, if any, is given by $\nu_{g,t}$. The supply of capital in period t, K_t, satisfies

$$K_t = \sum_{g=1}^{G} a_{g,t-1}. \tag{6}$$

If the bond market is operating, bonds are in zero net supply, hence for all t,

$$\sum_{g=1}^{G} \nu_{g,t} a_{g,t} = 0. \tag{7}$$

3.4 Government

Social Security benefits are financed by a wage tax, τ, and provided to all retirees on a per-capita basis. Let $H_{g,t}$ denote taxes and benefits paid and received by agents who are age g at time t. Then

$$H_{g,t} = \tau w_t \ell_g \tag{8}$$

and

$$B_{g,t} = (1 - \ell_g) \frac{\sum_{j=1}^{G} H_{j,t}}{80 - R}, \tag{9}$$

where (9) enforces Social Security budget balance.

3.5 Households

At time t, the economy's state is $(s_t,\, z_t)$, with $s_t = (x_{1,t},\ldots,x_{G-1,t})$ denoting the set of age-specific holdings of cash-on-hand.[5] Households of age g in state (s, z) maximize expected

[5] Note that $x_{G,t}$, the cash-on-hand of the oldest generation is not included in the state vector, because it can be inferred from the other state variables.

remaining lifetime utility given by

$$V_g(s,z) = \max_{c,a,\nu} \left\{ u(c) + \beta E\left[V_{g+1}(s',z')\right] \right\} \quad \text{for} \quad g < G, \text{ and} \tag{10}$$

$$V_G(s,z) = u(c) \tag{11}$$

subject to

$$c_{1,t} = \ell_1 w_t - a_{1,t} - H_{1,t} + B_{1,t}, \tag{12}$$

$$\begin{aligned} c_{g,t} = {} & \ell_g w_t + \left[\nu_{g-1,t-1}(1+\bar{r}_{t-1}) + (1-\nu_{g-1,t-1})(1+r_t)\right] a_{g-1,t-1} - a_{g,t} \\ & - H_{g,t} + B_{g,t}, \end{aligned} \tag{13}$$

for $1 < g < G$, and

$$\begin{aligned} c_{G,t} = {} & \ell_G w_t + \left[\nu_{G-1,t-1}(1+\bar{r}_{t-1}) + (1-\nu_{G-1,t-1})(1+r_t)\right] a_{G-1,t-1} \\ & - H_{G,t} + B_{G,t}, \end{aligned} \tag{14}$$

where $c_{g,t}$ is the consumption of a g-year old at time t and (12)–(14) are budget constraints for age group 1, those between 1 and G, and for those age G.

3.6 Equilibrium

Given the initial state of the economy $(x_0, z_0) = (x_{1,0}, \ldots, x_{G-1,0}, z_0)$, the recursive competitive equilibrium is defined as follows.

Definition. The recursive competitive equilibrium is governed by the consumption functions, $c_g(s,z)$, the share of savings invested in bonds, $\nu_g(s,z)$, factor demands of the representative firm, $K(s,z)$ and $L(s,z)$, government policy, $H(s,z)$ and $B(s,z)$, as well as the pricing functions, $r(s,z)$, $w(s,z)$, and $\bar{r}(s,z)$, such that

1. Given the pricing functions, the value functions (10) and (11) solve the recursive problem of the households subject to the budget constraints (12)–(14), and a_g, ν_g, and c_g

are the associated policy functions for all g and for all dates and states.

2. Wages and rates of return on capital satisfy (3) and (4).

3. The government budget constraint (9) is satisfied.

4. All markets clear.

5. Finally, for all age groups $g = 1, \ldots, G-1$, optimal intertemporal consumption and investment choice satisfy

$$1 = \beta E_z\big[\big(1 + r(s', z')\big)\frac{u'(c_{g+1}(s', z'))}{u'(c_g(s, z))}\big] \tag{15}$$

and

$$0 = E_z\left[u'(c_{g+1}(s', z'))\big(\bar{r}(s, z) - r(s', z')\big)\right], \tag{16}$$

where E_z is the conditional expectation of z' given z (also taken over the i.i.d. depreciation shock δ).

4 Calibration

Our calibration is standard. We exclude demographic and secular technological change, since our focus is on risk, not long-term trends. We provide more details below.

4.1 Endowments and Preferences

Recall that agents work for 45 periods and live for 80. We set the quarterly subjective discount factor, β, to 0.99. This implies an annual value of 0.96 for β. Risk aversion, γ, equals 2.

4.2 Technology

TFP Shocks

Our assumed quarterly values for ρ and σ are 0.95 and 0.01, respectively.[6] On an annual basis, they are 0.814 and 0.019, respectively, generating a mean TFP value of 0.997 with a standard deviation of 0.033 and a coefficient of variation of 3.29 percent. The resulting correlation coefficient between the wage and the rate of return on capital is 0.544.

Depreciation Shocks

One of our calibrations features both TFP and depreciation shocks. The TFP process is as above. The quarterly value of the standard deviation, ψ, of the depreciation shock, δ, is set at 0.011, implying an annual value of 0.045.[7] This is far higher than the 0.0052 quarterly estimate of Ambler and Paquet (1994). Recall that ψ is set to reproduce the standard deviation of the return to national wealth.

$$r_t = \frac{W_{t+1} - W_t + C_t - E_t}{W_t}, \tag{17}$$

where W_t, E_t, C_t, and r_t stand for time-t wealth, labor income, economy-wide consumption, and the rate of return on economy-wide wealth.[8]

Under this calibration, the correlation coefficient between the wage and the rate of return on capital is -0.054, well within Davis and Willen (2000) empirical estimates.

[6] Hansen (1985) estimates a quarterly value for the autocorrelation coefficient, ρ, in the TFP process of 0.95 and a standard deviation, σ, of the innovation ϵ ranging from 0.007 to 0.01. Prescott (1986) estimates are 0.9 for ρ and 0.00763 for σ.

[7] We treat equation (2) for Y as the net production function and set the mean value of depreciation at zero.

[8] We measure total labor income assuming labor's share of proprietorship and partnership income is the same as its share of national income. Note, some components of the Federal Reserve wealth series are carried at book, which biases downward our estimate of the variability of the national wealth return.

4.3 Government

We set the payroll tax rate, τ, at 15 percent. This is a reasonable value for the U.S. given that its Social Security and Medicare employee plus employer FICA tax is 15.3 percent.

5 Solution Method

Our solution method is that used by Hasanhodzic and Kotlikoff (2018, 2025b), which we recapitulate here. We start by drawing a sequence of aggregate shocks. Second, we guess consumption functions for each of our 79 generations as linear polynomials of the economy's state vector (the 80th generation just consumes their cash on hand). Third, we project the economy forward for 830 years from its initial conditions.[9] This involves clearing the bond market if one is assumed. Fourth, we use the model's Euler conditions to update our guessed decision functions. And fifth, we repeat steps two through four until the Euler conditions are satisfied to a high degree of precision.

More specifically, our algorithm contains outer and inner loops. The outer loop solves for consumption functions of each generation. This is GSSA. The inner loop uses a combination of techniques from the numerical analysis literature – Broyden, Gauss-Seidel, and Newton's method – to compute the agents' bond holdings and the risk-free rate that clears the bond market.

Recall that the state vector consists of cash-on-hand variables, $x_{g,t}$, of generations 1 through $G-1$ and the lagged TFP shock. Given the information at time t, agents decide how much of their cash on hand to consume, $c_{g,t}$. They also choose the proportion $\nu_{g,t}$ of their savings to allocate to bonds at the prevailing risk-free rate $\bar{r}_t$.

The outer loop starts by making an initial guess of generation-specific consumption functions c_g as linear polynomials in the state vector and the prevailing depreciation shock.[10]

[9] Longer simulation periods produce no differences in results.

[10] Although we do not include δ as part of the theoretical state space, using it as a regressor for approximating the consumption functions proved valuable.

Next, we take a draw of the path of shocks for T periods. We then run the model forward for T periods using the economy's initial condition (i.e., the state vector of age specific cash on hand holdings that arise in the long run with zero economic shocks), guessed consumption functions and the drawn shocks. I.e., we compute cash-on-hand variables at time $t+1$ using the information we have at time t and the exogenous shocks at time $t+1$. Since the ν's and the $\bar{r}$ that are determined at time t are realized at time $t+1$, their knowledge is necessary to compute cash-on-hand variables in period $t+1$.

In running the model forward, at each time t, we compute agents' holdings of bonds and the risk-free rate that clears the bond market. To solve for $\bar{r}_t$, we use Broyden's method based on the bond-market clearing condition. This condition requires that the sum of bond holdings at time t equals zero. The bond holdings at time t of each agent age g is $\nu_{g,t}a_{g,t}$. The choice of the $\nu_{g,t}$'s make them functions of $\bar{r}_t$. Hence, for given values of the $a_{g,t}$'s, the bond-market clearing condition is a function of $\bar{r}_t$ and can be used, via Broyden's method, to find the $\bar{r}_t$ that sustains market clearing.

For any given $\bar{r}_t$, the choice of $\nu_{g,t}$'s is determined by Gauss-Seidel iterations to solve the system of simultaneous $G-1$ generation-specific Euler equations governing the choices of the $G-1$ ν's for the new values of those ν's. Specifically, for given guesses of each agent's value of ν, other than that of agent i, we apply Newton's method to agent's i's Euler equation to determine the new guessed value of ν for agent i.[11]

Simulating the model forward produces the data needed to update our guessed consumption functions. Specifically, for each age group g and each period t, we evaluate the Euler condition to determine what that age group's consumption should be in that period. This calculation is based on the derived period-t state variables and the current guessed consumption functions of all agents, which enter, via their impact on the state vector of cash of hand at $t+1$, into the etermination of any given age-g agent's marginal utility of consumption at $t+1$. The expected value is evaluated using Gaussian quadrature, as in GSSA.

[11] Taking other unknowns as given is Gauss-Seidel.

Following PEA/GSSA, we then regress these time series of age-specific consumption levels on the state variables plus the depreciation shock and use the new regression estimates to update, with dampening, the polynomial coefficients of each guessed consumption function. We iterate the updating of these functions based, always, on the same draw of the path of shocks and continue until consumption functions converge.

We evaluate the accuracy of our solutions using two methods proposed in the literature – out-of-sample deviations from the exact satisfaction of the Euler equations and the statistic proposed by Den Haan and Marcet (1989, 1994).

5.1 Out-of-Sample Deviations from the Perfect Satisfaction of Euler Equations

A satisfactory solution requires that generation-specific Euler equations (15) hold out of sample. Hence, to test the accuracy of our solution, we draw a fresh sequence of 1660 sets of shocks for each simulated model. We then run the model forward for 1660 years (twice the length of the original simulation), imposing the drawn shocks, using the original consumption functions, c_g, and clearing the bond market by rerunning the model's inner loop each year as we move through time. To calculate out-of-sample, unit-free deviations from full satisfaction of the Euler equations, we form

$$\epsilon(s,z) = \beta E_z\big[\big(1 + r(s',z')\big)\frac{u'(c_{g+1}(s',z'))}{u'(c_g(s,z))}\big] - 1 \tag{18}$$

for each period in the newly simulated time path and for each generation $g \in 1,\ldots,G-1$. Finally, we compute the average, across time, of the absolute value of the deviations from these Euler equations for each generation.[12] In all cases these deviations are at most 0.003.

[12]Note, these deviations are not Euler errors, which capture differences in period t's marginal utility and period $(t+1)$'s realized marginal utility (properly weighted by β and $r(s',z')$). Rather, they reference mistakes in satisfying the Euler equation, i.e., the discrepancy in period t between the marginal utility and

In most cases, they are zero to the third decimal place.

The portfolio choice equations (16) and the bond market-clearing condition (7) hold almost perfectly by construction, since the ν's and $\bar{r}$ that satisfy them are calculated in the inner loop with a high degree of precision. In particular, the average absolute deviations from these equations, which theoretically should equal zero, are at most 0.0005 and 0.00001, respectively, and in most cases equal zero to the seventh decimal place.

5.2 The Den Haan-Marcet Statistic

An alternative precision test is provided by Den Haan and Marcet (1989, 1994). Taylor and Uhlig (1990) use this test to compare alternative solution methods for nonlinear stochastic growth models.

As above, we start with a fresh draw of shocks over T periods and simulate the model forward based on these shocks, using the original consumption functions and clearing the bond market each period based on the inner loop technique (discussed above). We set T again to 1660. Then, for each generation-specific Euler equation (16), we compute the residual, η_g, where g references the generation's age at time t.

$$\eta_g(t) = \beta\big(1 + r(t+1)\big)\frac{u'(c_{g+1}(t+1))}{u'(c_g(t))}. \tag{19}$$

We next regress, separately for each generation, their 1600 η_g values on a matrix x_g consisting of a constant, five lags of c_g, and five lags of z. The predicted values of the regression equation, $\hat{a}_g$,

$$\hat{a}_g = \big(\Sigma x_g(t)' x_g(t)\big)^{-1}\big(\Sigma x_g(t)' \eta_g(t)\big), \tag{20}$$

are then used to construct the Den Haan-Marcet statistic m_g as follows:

$$m_g = \hat{a}_g'\big(\Sigma x_g(t)' x_g(t)\big)\big(\Sigma x_g(t)' x_g(t)\eta_g(t)^2\big)^{-1}\big(\Sigma x_g(t)' x_g(t)\big)\hat{a}_g. \tag{21}$$

its properly weighted time-t expectation.

If the generation-specific Euler equations (16) are satisfied, then $E_{t-1}[\eta_g(t)] = 0$ must hold. This implies that the coefficient vector, and, therefore, m_g is zero, which is the null hypothesis. Note that our solution method does not enforce this property, so as Den Haan and Marcet (1994) point out, theirs is a challenging test.

Under the null, m_g is distributed as $\chi^2(11)$ asymptotically. Based on a two-sided test at the 2.5 percent significance level, we would fail to reject the null if m_g lies outside the interval (3.82, 21.92). For each of our models, we compute the minimum, mean, and maximum across generations of generation-specific statistics m_g. The mean across generations of the statistic is well within the acceptance interval for all models.

5.3 Algorithm

The following is a step-by-step description of our algorithm.

Initialization:

- Set $\bar{z}$ and $\bar{\delta}$ to their average values and solve for the deterministic steady state cash on hand of each age group without bond, $\bar{s} = (\bar{x}_1, \ldots, \bar{x}_{G-1})$. Note that the deterministic steady state corresponds to the state vector of age specific cash on hand holdings that arise in the long run with zero economic shocks. Let $(s_0, z_0, \delta_0) = (\bar{s}, \bar{z}, \bar{\delta})$ be the starting point of the simulation. In this initial condition, bond holdings are set to zero.

- Approximate $G - 1$ consumption functions by polynomials in the state variables and the shock delta: $c_1(s, z) = \phi_1(s, z, \delta; b_1), \ldots, c_{G-1}(s, z) = \phi_{G-1}(s, z, \delta; b_{G-1})$, where $b_1, \ldots, b_{G-1}$ are polynomial coefficients. We use linear polynomials. To start the iterations, we make the following initial guess for the coefficients:
  $b_1 = (0, 0.9\bar{c}_1/\bar{x}_1, 0, \ldots, 0, 0.1\bar{c}_1, 0), \ldots, b_{G-1} = (0, 0, \ldots, 0, 0.9\bar{c}_{G-1}/\bar{x}_{G-1}, 0.1\bar{c}_{G-1}, 0)$.

- Take draws of the exogenous path of shocks for T - 830 years.

Outer loop:

- The first step in the outer loop is to simulate the model forward, i.e. compute the state space for $t = 1, \ldots, T$. To do so, at each time t we proceed as follows:
  - Recall that at time t, the state vector consists of the vector of cash-on-hand variables of generations 1 through $G-1$, $s_t = (x_{1,t}, \ldots, x_{G-1,t})$ and exogenous shocks.
  - Using this state vector, for each age group g, calculate its consumption $c_g^{(p)}$ given the current guess of the coefficients $b_g^{(p)}$, where the subscript (p) denotes the current iteration of the outer loop. I.e., $c_{g,t}^{(p)}$ equals the inner product of the vector $(1, s_t, z_t, \delta_t)$ with the vector of coefficients $b_g^{(p)}$. Compute the generation-specific asset demands, $a_{g,t}$, as the difference between cash on hand and consumption, $a_{g,t} = x_{g,t} - c_{g,t}$. Note that the sum of asset demands of generations 1 through $G-1$ is the capital stock at the beginning of period $t+1$, k_{t+1}.
  - At this point enter the inner loop to compute the agents' choices of bond shares, $\nu_{g,t}$, for generations 1 through $G-1$, and the risk free rate $\bar{r}_t$. Recall that these are needed to compute the cash-on-hand variables at time $t+1$.
  - Inner loop:
    * Use Broyden's method to solve (7) for $\bar{r}_t$. To start, make an (arbitrary) initial guess for the value of $\bar{r}_t$.
    * Given $\bar{r}_t$, solve the system of $G-1$ equations given by (17) for $g = 1, \ldots, G-1$, for $G-1$ unknowns, $\nu_{1,t}, \ldots, \nu_{G-1,t}$. To do so, approximate the expectation by Gaussian quadrature.[13] Notice that the consumption at time $t+1$, $c_{g,t+1}$, on the right-hand-side of each equation (17) needs to be approximated by the polynomial in the state vector plus δ. Hence, each of these equations depends

[13] We use 4 nodes in the quadrature, using more does not change the results.

on the entire distribution of the cash-on-hand variables, and through them, on all of the generation-specific ν's, $\nu_{1,t}, \ldots, \nu_{G-1,t}$. To solve a nonlinear system of $G-1$ nonlinear equations in $G-1$ unknowns we use the Gauss-Seidel algorithm, which reduces the problem of solving for $G-1$ unknowns simultaneously in $G-1$ equations to that of iteratively solving $G-1$ equations in one unknown.[14] We solve each of these nonlinear equations in one unknown ν using Newton's method.

  * Use $\nu_{g,t}$ found above for all g to calculate (7) and update $\bar{r}_t$.

- Given $\nu_{g,t}$ for $g = 1, \ldots, G-1$, $\bar{r}_t$, k_{t+1}, and exogenous shocks, we can now compute each generation's cash on hand in period t+1 as the sum of their labor and capital income (plus or minus any government transfers) at time t+1.

- Note that for each age group g and each state (s_t, z_t), $t = 1, \ldots, T$, (16) implies

$$c_g(s, z) = \left\{\beta E_z\left[\left(1 + r(s', z')\right)u'(c_{g+1}(s', z'))\right]. \qquad (22)$$

  Denote the right-hand-side of (22) by y_g and evaluate the expectation using Gaussian quadrature.

- For each age group g, regress y_g on (s_t, z_t, δ_t) and a constant term using regularized least squares with Trikhonov regularization. Denote the estimated regression coefficients by $\hat{b}_g^{(p)}$.

- Check for convergence: If

$$\frac{1}{G-1}\sum_{g=1}^{G-1}\frac{1}{T}\sum_{t=1}^{T}\left|\frac{x_g^{(p-1)} - x_g^{(p)}}{x_g^{(p-1)}}\right| < \epsilon,$$

[14]As the starting point for Gauss-Seidel we use the ν's computed at time t-1.

end. Otherwise, for each age group g update the coefficients as $b_g^{(p+1)} = (1-\xi)b_g^{(p)} + \xi\hat{b}_g^{(p)}$ and return to the beginning of the outer loop. We use $\xi = 0.1$ and $\epsilon \in [10^{-7}, 10^{-13}]$.

6 Results

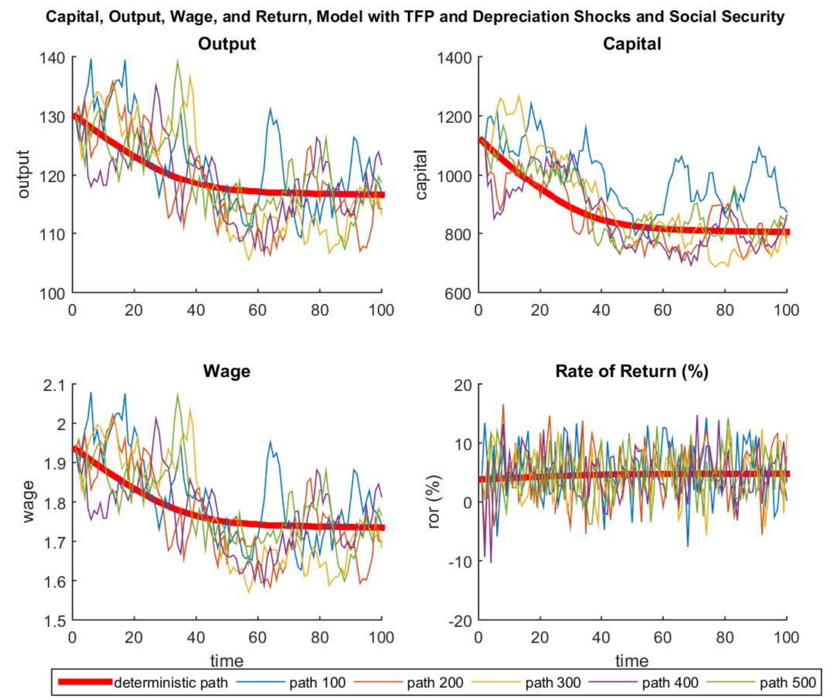

Figure 1: Evolution of output, capital, the wage, and the rate of return in the model with both TFP and depreciation shocks (including Social Security) for five illustrative shock paths. The corresponding deterministic path is shown for comparison. The figure displays the first 100 years.

Figure 1 plots the evolution of output, capital, the wage, and the rate of return for five representative stochastic paths in the model with both TFP and depreciation shocks and Social Security, together with the corresponding deterministic transition path for comparison. The figure reports the first 100 years. Although the stochastic paths exhibit short-run fluctuations, they cluster tightly around the deterministic trajectory notwithstanding the high serial correlation of the TFP shocks. Output, wages, and capital remain remarkably close to their deterministic counterparts, and even the rate of return – while visibly more volatile – shows no meaningful long-run departure from the deterministic path. These patterns underscore that aggregate shocks generate only modest, transitory variations and leave the economy's transitional dynamics essentially unchanged.

Fraction of Absolute Deviations from Deterministic Counterparts (in Percent)

	Threshold				Threshold			
	5%	10%	25%	50%	5%	10%	25%	50%
Years Out	**OUTPUT**				**CAPITAL**			
	TFP Shocks Only				**TFP Shocks Only**			
50	20.10	1.03	0	0	23.48	1.52	0	0
75	20.39	1.28	0	0	26.12	2.39	0	0
100	20.38	1.18	0	0	25.48	2.67	0	0
	TFP and Depreciation Shocks				**TFP and Depreciation Shocks**			
50	41.44	9.82	0	0	20.10	1.03	0	0
75	40.49	9.54	0	0	20.39	1.28	0	0
100	40.97	9.43	0	0	20.38	1.18	0	0
	WAGE				**RATE OF RETURN**			
	TFP Shocks Only				**TFP Shocks Only**			
50	20.10	1.03	0	0	14.56	0.53	0	0
75	20.39	1.28	0	0	16.24	0.51	0	0
100	20.38	1.18	0	0	16.31	0.58	0	0
	TFP and Depreciation Shocks				**TFP and Depreciation Shocks**			
50	41.44	9.82	0	0	96.09	91.77	79.63	60.16
75	40.49	9.54	0	0	95.63	91.18	78.94	60.60
100	40.97	9.43	0	0	95.98	91.53	79.47	60.52

Table 1: Fraction of absolute percentage deviations from deterministic benchmark values that exceed thresholds of 5, 10, 25, and 50 percent. The table reports results at horizons of 50, 75, and 100 years. Fractions are computed across 10,000 simulated histories of aggregate shocks. All simulations are based on model variants that include Social Security.

Table 1 shows the fractions of absolute percentage deviations from deterministic benchmark values that exceed thresholds of 5, 10, 25, and 50 percent. The table reports results at horizons of 50, 75, and 100 years, based on 10,000 simulated histories of aggregate shocks. All simulations use model variants that include Social Security. The results indicate that, for output, capital, and the wage, deviations from deterministic values are generally modest: for example, assuming only TFP shocks, roughly 20 percent of simulations produce deviations exceeding 5 percent, while deviations exceeding 10 percent occur in only about 1 percent of cases, and none exceed 25 percent. Adding depreciation shocks increases dispersion in the short run—especially for capital, where the share above the 5 percent threshold roughly doubles. But even then, there are no deviations above 25 percent across different horizons.

By contrast, the rate of return exhibits visibly greater sensitivity to aggregate shocks. With both TFP and depreciation shocks, more than 90 percent of simulated paths produce deviations exceeding 10 percent, and a substantial fraction exceed 25 percent and 50 percent at all horizons. This pattern aligns with the visual evidence from the time-series plots – returns react sharply to short-run fluctuations in productivity and capital depreciation, whereas the levels of output, capital, and the wage remain highly stable.

Overall, the table reinforces the conclusion that, from a long-run perspective, macro shocks have limited quantitative impacts on the evolution of aggregate variables with the exception of the rate of return.

Fraction of Absolute Consumption Compensating Differentials Above the Treshold (in Percent)

Model	Treshold		
	5%	10%	25%
TFP Shocks Only	0.5	0	0
TFP and Depreciation Shocks	34.5	5.6	0

Table 2: Fraction of compensating consumption differentials exceeding 5, 10, and 25 percent. Fractions are computed over 1,000 simulated shock paths in models with (i) TFP shocks only and (ii) both TFP and depreciation shocks. For each shock path, we calculate the realized lifetime utility of an individual born in year 50 and determine the compensating consumption factor λ that equates this utility to that of a newborn in the corresponding deterministic economy. Models include Social Security.

Table 2 reports the fraction of compensating consumption differentials whose absolute deviation from one exceeds 5, 10, and 25 percent. We consider two versions of the model – one with TFP shocks only and one with both TFP and depreciation shocks – and both specifications include Social Security, so differences across rows isolate the incremental effect of depreciation shocks. For each of 1,000 simulated shock paths, we evaluate the realized lifetime utility of an individual born in year 50 and compute the compensating consumption factor λ needed to equate this utility with that of a newborn in the corresponding deterministic economy. Because we work with $|\lambda - 1|$, a value exceeding 5 percent indicates that lifetime consumption would need to be scaled either up or down by more than 5 percent to offset the stochastic deviation relative to the deterministic benchmark.

The results show that large absolute deviations are rare. With TFP shocks only, just 0.5 percent of simulations produce $|\lambda - 1| > 0.05$, and none exceed 0.10 or 0.25. Introducing depreciation shocks increases dispersion, with 34.5 percent of paths yielding $|\lambda - 1| > 0.05$, and 5.6 percent exceeding 0.10, but no realizations surpassing 0.25. Even when both aggregate shocks are present, the required compensating adjustments remain small – only a minor upward or downward scaling of lifetime consumption is needed to replicate deterministic-path utility. This further underscores the second-order nature of aggregate shocks in our framework.

Table 3 compares the volatility of output, consumption, and investment in the model with their empirical counterparts. In the data, the standard deviation of detrended real NNP is 3.26 percent, while consumption and investment exhibit similar volatility, of 3.05 percent and 3.17 percent, respectively. In the model with only TFP shocks, the volatilities of all three variables are comparable to those observed in the data. Allowing for both TFP and depreciation shocks increases the volatility of all three aggregates, though the model continues to match the data reasonably well. The table also shows that the presence or absence of a bond market is largely irrelevant to the aggregate behavior described above.

Table 4 shows that introducing a bond market has virtually no effect on aggregate out-

Standard Deviation of Percent Deviation from Trend, Models vs. Data

Model	Standard Deviation (%)		
	Output	Cons	Inv
TFP Shocks Only			
No Bonds			
No Social Security	3.73	2.67	2.68
Social Security	3.82	2.48	2.48
With Bonds			
No Social Security	3.69	2.53	2.53
Social Security	3.79	2.41	2.41
TFP and Depreciation Shocks			
No Bonds			
No Social Security	5.63	5.87	5.82
Social Security	5.61	4.98	4.94
With Bonds			
No Social Security	6.06	5.05	5.00
Social Security	6.01	4.41	4.38
Data			
Data	3.26	3.05	3.17

Table 3: Standard deviation of percent deviations from trend for annual U.S. data – real NNP (1929–2024), real personal consumption expenditures (1929–2024), and real gross domestic investment (1967–2024) – and the standard deviation of percent deviations of output, aggregate consumption, and investment from their means in the model. For the data (model) calculations, consumption and investment are divided by real NNP (output) prior to detrending. The first 350 observations (out of 830) are discarded to ensure that the calculations reflect the stochastic steady state.

comes. The moments of output, capital, wages, and returns are nearly identical with and without bonds.[15] As argued by Hasanhodzic and Kotlikoff (2025a), households are effectively indifferent between holding bonds and capital, and the bond market plays little role in shaping macroeconomic outcomes. Comparing the rate of return on capital and the risk-free rate shows that the equity premium is virtually zero, confirming that the equity-premium puzzle extends to RBC in OLG models.

[15]The corresponding time paths, analogous to those presented in Figure 1, are visually indistinguishable with and without the bond market.

Means and Standard Deviations of Aggregate Variables in Models With and Without Bonds

Model	Means					Standard Deviations				
	Output	Capital	Wage	RoR on Capital (in %)	Risk-Free Rate	Output	Capital	Wage	RoR on Capital (in %)	Risk-Free Rate
NO BONDS										
TFP Shocks Only										
No Social Security	130.44	1124.47	1.94	3.83	-	4.87	0.07	40.05	0.13	-
Social Security	117.01	808.78	1.74	4.77	-	4.47	0.07	32.10	0.17	-
TFP and Depreciation Shocks										
No Social Security	128.34	1092.07	1.91	3.86	-	7.22	0.11	141.12	4.62	-
Social Security	114.87	779.94	1.71	4.85	-	6.45	0.10	99.28	4.63	-
WITH BONDS										
TFP Shocks Only										
No Social Security	130.33	1121.57	1.94	3.83	3.83	4.80	0.07	35.07	0.13	0.10
Social Security	116.62	800.43	1.74	4.80	4.80	4.42	0.07	29.06	0.17	0.14
TFP and Depreciation Shocks										
No Social Security	127.68	1076.67	1.90	3.91	3.74	7.74	0.12	157.07	4.63	0.41
Social Security	114.34	770.03	1.70	4.91	4.75	6.87	0.10	109.35	4.64	0.50

Table 4: Means and standard deviations of aggregate variables in models with and without bonds. The first 350 observations (out of 830) are discarded to ensure that the calculations reflect the stochastic steady state.

Finally, as reported in Hasanhodzic and Kotlikoff (2018, 2025b), the standard deviation of the return to national wealth is 4.89 percent in the data. Table 4 shows that, with TFP shocks only, the model generates values between 0.13 and 0.17 percent. When depreciation shocks are added, this rises to between 4.62 and 4.64 percent. Thus, while TFP shocks alone cannot match the volatility of risky returns, adding depreciation shocks brings the model in line with the data for risky returns. As Table 3 shows, this comes at the expense of overstating the variability of aggregate variables. However, this still fails to resolve the equity-premium puzzle.

7 Conclusion

This paper demonstrates that when macroeconomic shocks are calibrated to empirical data, their impact on long-run outcomes and lifetime welfare is very limited. Utilizing a large-scale stochastic OLG model with realistic TFP and depreciation shocks, we find that fluctuations in output, consumption, and investment remain modest. Furthermore, the probability of long-run aggregates meaningfully diverging from their deterministic steady-state values is negligible. In fact, differences in realized lifetime utility across cohorts remain small, even positing persistent shocks, confirming that macroeconomic risk is second-order. These results elucidate why the model generates a negligible equity premium and why bond markets have little impact on the macroeconomy, on economic welfare, or on the distribution of aggregate behavior. Our findings suggest that policies aimed at smoothing temporary fluctuations offer limited welfare gains. In contrast, policies that significantly redistribute across generations can materially impact the intergenerational distribution of welfare. But these impacts will be essentially identical whether or not the economy experiences macro shocks of realistic size and duration.

References

Alvarez, F. and U. J. Jermann, 2004, "Using Asset Prices to Measure the Cost of Business Cycles", *Journal of Political Economy* 112, 1223–1256.

Ambler, S. and A. Paquet, 1994, "Stochastic Depreciation and the Business Cycle", *International Economic Review* 35, 101–116.

Auerbach, A. J. and L. J. Kotlikoff, 1987, *Dynamic Fiscal Policy*, Cambridge, UK; New York, NY: Cambridge University Press.

Blundell, R., L. Pistaferri, and I. Preston, 2008, "Consumption Inequality and Partial Insurance", *American Economic Review* 98, 1887–1921.

Brumm, J., X. Feng, L. J. Kotlikoff, and F. Kubler, 2022, "Deficit Follies", *Journal of Political Economy* 130, 3145–3193.

Brumm, J., X. Feng, L. J. Kotlikoff, and F. Kubler, 2024, "When Interest Rates Go Low, Should Public Debt Go High?", *American Economic Journal: Macroeconomics* 16, 432–469.

Campbell, J. Y. and J. H. Cochrane, 1999, "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior", *Journal of Political Economy* 107, 205–251.

Constantinides, G. M. and D. Duffie, 1996, "Asset Pricing with Heterogeneous Consumers", *Journal of Political Economy* 104, 219–240.

Cooley, T. F. and E. C. Prescott, 1995, "Economic Growth and Business Cycles", in T. F. Cooley, editor, "Frontiers of Business Cycle Research", Princeton, NJ: Princeton University Press, 1–38.

Davis, S. J. and P. Willen, 2000, "Using Financial Assets to Hedge Labor Income Risks: Estimating the Benefits", Working paper, University of Chicago, Graduate School of Business.

Den Haan, W. J. and A. Marcet, 1989, "Accuracy in Simulations", CEPR Discussion Paper 181, Center for Economic Policy Research.

Den Haan, W. J. and A. Marcet, 1994, "Accuracy in Simulations", *The Review of Economic Studies* 61, 3–17.

Dolmas, J., 1998, "Welfare Costs of Business Cycles in a Representative-Agent Economy with Household Production", *Review of Economic Dynamics* 1, 646–676.

Gomme, P., R. Rogerson, P. Rupert, and R. Wright, 2005, "The Business Cycle and the Life Cycle", *NBER Macroeconomics Annual 2004* 19, 415–461.

Green, J. and L. J. Kotlikoff, 2008, "On the General Relativity of Fiscal Language", in A. J. Auerbach and D. N. Shaviro, editors, "Institutional Foundations of Public Finance", Cambridge, MA: Harvard University Press, 241–256.

Guvenen, F., S. Ozkan, and J. Song, 2014, "The Nature of Countercyclical Income Risk", *Journal of Political Economy* 122, 621–660.

Hansen, G. D., 1985, "Indivisible labor and the business cycle", *Journal of Monetary Economics* 16, 309–327.

Harenberg, D. and A. Ludwig, 2015, "Social Security in an Analytically Tractable Overlapping Generations Model with Aggregate and Idiosyncratic Risks", *International Tax and Public Finance* 22, 579–603.

Harenberg, D. and A. Ludwig, 2019, "Idiosyncratic risk, aggregate risk, and the welfare effects of social security", *International Economic Review* 60, 661–692.

Hasanhodzic, J. and L. J. Kotlikoff, 2018, "Generational Risk—Is It a Big Deal? Simulating an 80-Period OLG Model with Aggregate Shocks", Working Paper, NBER Working Paper 19179.

Hasanhodzic, J. and L. J. Kotlikoff, 2025a, "'No One Can Borrow' – Reassessing the 'Junior Can't Borrow' Equity-Premium Puzzle Resolution", NBER Working Paper 34328, National Bureau of Economic Research.

Hasanhodzic, J. and L. J. Kotlikoff, 2025b, "Studying Generational Risk in a Large-Scale Life-Cycle Model", Journal of Money, Credit and Banking Forthcoming.

Heathcote, J., K. Storesletten, and G. L. Violante, 2009, "Quantitative Macroeconomics with Heterogeneous Households", *Annual Review of Economics* 1, 319–354.

Judd, K. L., L. Maliar, and S. Maliar, 2011, "Numerically stable stochastic simulation approaches for solving dynamic economic models", *Quantitative Economics* 2, 173–210.

Kaplan, G. and G. L. Violante, 2010, "How Much Consumption Insurance beyond Self-Insurance?", *American Economic Journal: Macroeconomics* 2, 53–87.

Krusell, P. and A. Smith, 1998, "Income and Wealth Heterogeneity in the Macroeconomy", *Journal of Political Economy* 106, 867–896.

Lucas, R. E., 1987, *Models of Business Cycles*, Oxford, UK: Basil Blackwell.

Marcet, A., 1988, "Solution of nonlinear models by parameterizing expectations", Technical report, Carnegie Mellon University, Pittsburgh, PA.

Obstfeld, M., 1994, "Evaluating Risky Consumption Paths: The Role of Intertemporal Substitutability", *European Economic Review* 38, 1471–1486.

Otrok, C., 2001, "On Measuring the Welfare Cost of Business Cycles", *Journal of Monetary Economics* 47, 61–92.

Pallage, S. and M. A. Robe, 2003, "On the Welfare Cost of Business Cycles in Developing Countries", *International Economic Review* 44, 677–698.

Prescott, E. C., 1986, "Theory ahead of business-cycle measurement", *Carnegie-Rochester Conference Series on Public Policy* 25, 11–44.

Ríos-Rull, J.-V., 1994, "On the quantitative importance of market completeness", *Journal of Monetary Economics* 34, 462–496.

Ríos-Rull, J.-V. and R. Santaeulalia-Llopis, 2010, "Redistributive shocks and productivity shocks", *Journal of Monetary Economics* 57, 931–948.

Storesletten, K., C. Telmer, and A. Yaron, 2007, "Asset Pricing with Idiosyncratic Risk and Overlapping Generations", *Review of Economic Dynamics* 10, 519–548.

Taylor, J. B. and H. Uhlig, 1990, "Solving Nonlinear Stochastic Growth Models: A Comparison of Alternative Solution Methods", *Journal of Business & Economic Statistics* 8, 1–17.