Public Education Expenditure, Growth and Welfare*

Konstantinos Angelopoulos
University of Glasgow

Jim Malley
University of Glasgow and CESifo

Apostolis Philippopoulos
Athens University of Economics & Business, CESifo and University of Glasgow

September 4, 2007

Abstract

In this paper, we study the effects of public education expenditure on long-run growth, lifetime utility and transitional dynamics. The setup is a standard dynamic stochastic general equilibrium model, in which human capital is the engine of endogenous growth. Using comparable measures of human and physical capital, from Jorgenson and Fraumeni (1989, 1992a,b), we calibrate and solve the model for the USA economy in the post-war period. Our results suggest that while public spending on education can be both growth and welfare promoting, the latter can only be realized if increases in education expenditure are accompanied by changes in the government tax-spending mix.

*We would like to thank Fabrice Collard, Harris Dellas, Campbell Leith, Hamish Low, Vivek Ghosal, Arti Grover, Ioana Moldovan, Michael Moore, Efraim Sadka, Jonathan Temple, Klaus Wälde, Ulrich Woitek, Fabrizio Zilibotti and seminar participants at the Queens University Belfast, the Public Economics UK Conference and the CESifo conference on Productivity and Growth for helpful suggestions. The usual disclaimer applies.

1 Introduction

The path-breaking work of Romer (1986) and Lucas (1988), stressing the roles of knowledge and human capital accumulation, has led to an enormous body of theoretical and empirical literature attempting to better understand the determinants of endogenous (long-run) growth. The same literature has also emphasized the potential role of human capital externalities by allowing the return on the human capital of private agents to be increasing in the average human capital in the economy (see e.g. Lucas (1988), Azariadis and Drazen (1990) and Tamura (1991)).[1] On the other hand, there are economists who believe that general human capital externalities are not likely to be large (see e.g. Heckman and Klenow (1997) and Judd (2000)).

Although the magnitude of human capital externalities remains an open issue, and it is these externalities that typically justify public education policies, the latter are an economic and political reality. In almost all countries, governments follow a variety of public education policies. As a result, there is a growing theoretical literature on the effects of public education policies on growth and welfare (see e.g. Glomm and Ravikumar (1992), Zhang (1996), Blankenau and Simpson (2004), Su (2004), and Blankenau (2005)). Nevertheless, we are not aware of any estimation or calibration research which explores the empirical links between public education, growth and welfare. Accordingly, in this paper, we calibrate, solve and conduct policy analysis using a fairly standard dynamic stochastic general equilibrium model whose engine of long-term growth is human capital accumulation.

Our approach allows for positive externalities generated by the stock of average economy-wide human capital, as well as public education expenditure. Both the human capital externality and public education expenditure enter as inputs in private human capital accumulation. In other words, they can both enhance the productivity of households' private education choices. The way we model human capital externalities is as in e.g. Azariadis and Drazen (1990) and Tamura (1991), while the way we model public education expenditure follows e.g. Blankenau and Simpson (2004), Su (2004) and Blankenau (2005).

Our empirical base of departure for our model calibration is the post-war US economy. After we evaluate the ability of our model to replicate the

[1] The importance of externalities in this context is emphasised by Lucas (2002) who states that "if ideas are the engine of growth and if an excess of social over private returns is an essential feature of the production of ideas, then we want to go out of our way to introduce external effects into growth theory, not to try to do without them" and "the existence of important external effects of investment in human capital – in knowledge – has long been viewed as an evident and important aspect of reality".

main features observed in the US, we use it to shed light on the implications of changes in public education expenditure as a share of output for long-run growth, lifetime utility and the transition path to the steady-state. To be in a position to realistically assess the effects of public education expenditure, we assume that it is financed by a distorting tax on income. But we also study the cases in which the same changes in public education expenditure are financed by changes in lump-sum taxes/transfers (i.e. the benchmark case), as well as the case in which all types of government expenditure change by the same proportion. To solve the model and welfare evaluate different policies, we work as in Schmitt-Grohé and Uribe (2004, 2007) by approximating both the equilibrium solution and expected lifetime utility to second-order.

Our calibration profits significantly from having access to a dataset which includes consistent measures for human and physical capital (see e.g. Jorgenson and Fraumeni (1989)). In contrast to the relevant empirical studies referred to above, this data allows us to correctly distinguish between inputs to and output from the human capital production function.[2]

Our main findings are as follows. First, there is evidence of positive externalities from the average stock of human capital in the economy.[3] There is also evidence that public education expenditure augments private human capital accumulation.

Second, the growth and welfare effects of public education expenditure are not monotonic. Specifically, the pattern between public education expenditure and long-run growth, as well as the pattern between public education expenditure and lifetime utility, are Laffer curves with the peak of the curves giving the growth-maximizing and the welfare-maximizing public education expenditure as share of output. Given that higher public education spending crowds-out private consumption, the welfare maximising share is much smaller than the growth maximising one.

Third, substantial welfare gains are obtainable if increases in public education spending are accompanied by changes in the government tax-spending mix. In particular, welfare gains can be made if increases in public education

[2] Given the lack of comparable cross-country human capital data, other empirical work generally resorts to the use of measures of school enrolment ratios or years of schooling as general proxies of labor quality or human capital.

[3] There has been surprisingly little empirical work on the magnitude of human capital externalities at the aggregate level. Indeed, we are only aware of one study which attempts to directly estimate human capital externalities in a growth context (see Gong *et al.* (2004)), while e.g. Mamuneas *et al.* (2001) and Heckman and Klenow (1997) examine the link between externalities and the level of aggregate output. There has however been much more work at the sub-aggregate level (see the literature review in e.g. Ciccone and Peri (2006)). In general, it seems that in both the macro and micro literatures, the empirical size of externalities remains inconclusive.

spending are met by decreases in other components of government spending so as further distorting taxes are not required.

Fourth, increases in uncertainty (namely, the volatility of the innovations to the processes driving public education spending and total factor productivity) reduce expected lifetime utility, as well as the welfare gains associated with using lump-sum versus distorting taxes to finance public education expenditure.

Fifth, a positive public education spending shock generates a non-monotonic reaction in the growth rate and other key economic variables, so that after an initial boom there is a decrease in the growth rate below its steady-state value. The quantitative effects of discretionary fiscal policy of this form are not trivial as they are similar in magnitude to those of a standard TFP shock.

The rest of the paper is organized as follows. Section 2 summarizes the theoretical model. Section 3 discusses the data, calibration and model fit. Section 4 contains the results and Section 5 the conclusions. Finally, the Appendix presents further information on the deterministic steady-state and the second-order welfare function.

2 Theoretical Model

In this section, building on Lucas' (1990) model, we present and solve a DSGE model in which the engine of long-term growth is human capital accumulation.[4] To conduct our analysis, in comparison to the Lucas setup, we add externalities generated by the average stock of human capital in the society as well as government expenditure on public education. We also operate in a stochastic environment which allows us to explicitly analyse the effects of uncertainty on welfare.

We abstract from public subsidies to private education[5] and private education expenditure. We also assume away other common types of government spending (government investment in infrastructure and utility-enhancing public consumption) and use a relatively simple public finance structure with a single income tax and a lump-sum tax/subsidy instrument only.

The general equilibrium solution of the model consists of a system of dy-

[4] We choose Lucas' model because it is well known and its conclusions are rather robust (see Stokey and Rebelo (1995) for a quantitative comparison of some important DSGE models of human capital).

[5] We thus do not study the allocation of public funds between alternative uses, e.g. between basic public education and subsidies to private college education, which is the focus in e.g. Zhang (1996), Su (2004) and Blankenau (2005).

namic relations jointly specifying the paths of output, consumption, physical capital, the growth rate of human capital, the fractions of time allocated to work and education, and one residually determined policy instrument. To obtain these paths, we solve the second-order approximation of our model's equilibrium conditions around the deterministic steady-state (see also e.g. Schmitt-Grohé and Uribe (2004)). In contrast to solutions which impose certainty equivalence, the solution of the second-order system allows us to take account of the uncertainty agents face when making decisions. In addition, as pointed out by e.g. Schmitt-Grohé and Uribe (2004), the second-order approximation to the model's policy function helps to avoid potential spurious welfare rankings which may arise under certainty equivalence. In other words, when we evaluate different policies and regimes, we will approximate both the equilibrium solution and welfare (defined as the conditional expectation of lifetime utility) to second-order (see e.g. Schmitt-Grohé and Uribe (2007)). Further details are in subsection 4.1 below.

2.1 Households

The model is populated by a large number of identical households indexed by the subscript h and identical firms indexed by the subscript f, where $h, f = 1, 2, ..., N_t$. The population size, N_t, evolves at a constant rate $n \geq 1$, so that $N_{t+1} = nN_t$, where N_0 is given. Each household's preferences are given by the following time-separable utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^h) \tag{1}$$

where E_0 is the conditional expectations operator; C_t^h is consumption of household h at time t; and $0 < \beta < 1$ is the subjective rate of time preference. The instantaneous utility function is increasing, concave and satisfies the Inada conditions. We use the $CRRA$ form for utility:

$$U_t = \frac{\left(C_t^h\right)^{1-\sigma}}{1-\sigma} \tag{2}$$

where, $1/\sigma$ ($\sigma > 1$) is the intertemporal elasticity of substitution between consumption in adjacent periods.

Each household saves in the form of investment, I_t^h, and receives interest income, $r_t K_t^h$, where r_t is the return to private capital and K_t^h is the beginning-of-period private capital stock. Each household also has one unit of effort time in each period t, which it allocates to work, u_t^h and education,

e_t^h, so that $u_t^h + e_t^h = 1$[6]. A household with a stock of human capital, H_t^h receives labour income, $w_t u_t^h H_t^h$, where w_t is the wage rate and $u_t^h H_t^h$ is effective labour. Finally, each household receives dividends paid by firms, Π_t^h. Accordingly, the budget constraint of each household is

$$C_t^h + I_t^h = (1 - \tau_t) \left[r_t K_t^h + w_t u_t^h H_t^h + \Pi_t^h \right] + \overline{G}_t^o \tag{3}$$

where $0 < \tau_t < 1$ is the distortionary income tax rate and $\overline{G}_t^o$ is an average (per household) lump-sum tax/subsidy. The role played by $\overline{G}_t^o$ will be explained below in subsections 2.3 and 2.5.

Each household's physical and human evolve according to the following relations

$$K_{t+1}^h = (1 - \delta^p) K_t^h + I_t^h \tag{4}$$

and

$$H_{t+1}^h = (1 - \delta^h) H_t^h + \left(e_t^h H_t^h \right)^{\theta_1} \left(\overline{H}_t \right)^{1-\theta_1} \widetilde{B}_t \tag{5}$$

where, $0 \leq \delta^p, \delta^h \leq 1$ are constant depreciation rates on private physical and human capital respectively. The second expression on the r.h.s. of (5), consisting of three multiplicative terms, can be interpreted as the quantity of "new" human capital created at time period t. This expression is comprised of the following arguments: (i) $\left(e_t^h H_t^h \right)$ is $h's$ effective human capital; (ii) $\overline{H}_t$ is the average (per household) human capital stock in the economy; (iii) $\widetilde{B}_t \equiv B \left(g_t^e \right)^{\theta_2}$ represents human capital productivity, where $B > 0$ is a constant scale parameter and g_t^e is average (per household) public education expenditure expressed in efficiency units; (iv) the parameters $0 < \theta_1 \leq 1, 0 \leq (1 - \theta_1) < 1, 0 \leq \theta_2 < 1$ capture the productivity of household's human capital, the aggregate human capital externality and public education spending respectively.[7]

The assumption that individual human capital accumulation is an increasing function of the per capita level of economy-wide human capital encapsulates the idea that the existing know-how of the economy provides an external positive effect. Equivalently it can be thought of as a learning-by-doing effect as discussed in Romer (1986). Examples of other papers which use the per capita level of aggregate human capital in either the goods or human capital production functions include Lucas (1988), Azariadis and Drazen, (1990), Tamura (1991) and Glomm and Ravikumar (1992).

[6] We do not endogenize leisure since this generally leads to multiple long-run equilibria in this class of models with human capital (see e.g. Ortigueira, 1998, who also provides relevant references). Stokey and Rebelo (1995) discuss the implications of elastic labour supply in growth models with human capital.

[7] Following e.g. Lucas (1988), we assume that human capital is basically the only input in human capital accumulation.

The assumption that individual human capital accumulation depends on the per household public education share, g_t^e, in (5) is consistent with the goal of public education policy in practice as well as with theoretical work (see e.g. Glomm and Ravikumar (1992), Blankenau and Simpson (2004), Su (2004) and Blankenau (2005)).[8] Finally note that, the parameter restrictions employed in (5) imply increasing returns to scale (IRS) at the social level.[9]

Households act competitively by taking prices, policy variables and aggregate outcomes as given. Thus, each household h chooses $\{C_t^h, u_t^h, e_t^h, I_t^h, K_{t+1}^h, H_{t+1}^h\}_{t=0}^{\infty}$ to maximize (1) subject to (3), (4), (5), the time constraint $u_t^h + e_t^h = 1$, and initial conditions for K_0^h and H_0^h.

Substituting (4) into (3) for I_t^h and using the time constraint for u_t^h, we next derive the first order conditions. The familiar static optimality condition for consumption, C_t^h, is

$$\Lambda_t^h = \left(C_t^h\right)^{-\sigma} \tag{6}$$

and states that the shadow price associated with (3), Λ_t^h, is equal to the marginal value of consumption at time t.

The Euler-relation for private physical capital, K_{t+1}^h, is given by

$$\Lambda_t^h = \beta E_t \Lambda_{t+1}^h \left[1 - \delta^p + (1 - \tau_{t+1})\, r_{t+1}\right] \tag{7}$$

and reveals that the marginal cost of forgone consumption at time t is equal to the expected marginal benefit of discounted $t+1$ net (of tax) returns derived from investing in one unit of physical capital at time t.

The static optimality condition for time spent on education, e_t^h, can be written as

$$\Psi_t^h = \frac{\Lambda_t^h (1 - \tau_t)\, w_t H_t^h}{B\theta_1 \left(e_t^h\right)^{\theta_1 - 1} \left(H_t^h\right)^{\theta_1} \left(\overline{H}_t\right)^{1-\theta_1} \left(g_t^e\right)^{\theta_2}} \tag{8}$$

and says that the shadow price associated with (5), Ψ_t^h, is equal to the marginal value of education at time t.

The Euler-equation for private human capital, H_{t+1}^h, is

$$\begin{aligned} \Psi_t^h \;&=\; \beta E_t \Lambda_{t+1}^h (1 - \tau_{t+1})\, w_{t+1} u_{t+1}^h + \\ &\quad \beta E_t \Psi_{t+1}^h \left[\left[1 - \delta^h + B\theta_1 \left(e_{t+1}^h\right)^{\theta_1} \left(H_{t+1}^h\right)^{\theta_1 - 1} \left(\overline{H}_{t+1}\right)^{1-\theta_1} \left(g_{t+1}^e\right)^{\theta_2}\right]\right. \end{aligned} \tag{9}$$

[8] Blankenau (2005, pp. 493-4) also has a good discussion of the effects of public education on students' achievement. As he points out, assuming a positive effect is not uncontroversial, this is why public expenditures "are included with a parameter θ_2 to gauge their relative importance in producing human capital".

[9] Lucas (1988) and Benhabib and Perli (1994) are examples of other studies which employ the IRS assumption in either or both the physical and human capital production functions.

and expresses that the marginal cost of forgone labour income at time t is equal to the marginal benefit of discounted net (of tax) $t+1$ returns to working plus the marginal $t+1$ returns to investing in one unit of human capital at time t.

Finally, the transversality conditions

$$\lim_{t\to\infty}\left[\beta^t U_{C^h}(t) K_{t+1}^h\right]=0 \tag{10}$$

and

$$\lim_{t\to\infty}[\beta^t U_{C^h}(t) H_{t+1}^h]=0 \tag{11}$$

maintain and state that the discounted value of a household's physical and human assets must approach zero in the limit.

2.2 Firms

To produce its homogenous final product, Y_t^f, each firm, f, chooses private physical capital, K_t^f, and effective labour, $u_t^f H_t^f$. Thus, the production function of each firm is:

$$Y_t^f = A_t \left(K_t^f\right)^{\alpha_1}\left(u_t^f H_t^f\right)^{1-\alpha_1} \tag{12}$$

where A_t represents the level of Hick neutral neutral technology available to all firms, $0<\alpha_1,(1-\alpha_1)<1$ are the productivity of private capital and labour respectively.

Firms act competitively by taking prices, policy variables and aggregate outcomes as given. Accordingly, subject to (12), each firm f chooses K_t^f and $u_t^f H_t^f$ to maximize a series of static profit functions,

$$\Pi_t^f = Y_t^f - r_t K_t^f - w_t u_t^f H_t^f. \tag{13}$$

The resulting familiar first-order conditions state that the firm will hire labour until the marginal product of effective labour is equal to the wage rate, w_t, and will rent capital until the marginal product of physical capital is equal to the rental rate, r_t, i.e.

$$\frac{(1-\alpha_1)\,Y_t^f}{u_t^f H_t^f}=w_t \tag{14}$$

$$\frac{\alpha_1 Y_t^f}{K_t^f}=r_t. \tag{15}$$

2.3 Government

Total expenditure on public education, G_t^e and other lump-sum types of transfers/taxes, G_t^o, are financed by total income tax revenue. Thus,

$$G_t^e + G_t^o = \tau_t \sum_{h=1}^{N_t} \left(r_t^k K_t^h + w_t u_t^h H_t^h + \Pi_t^h \right) \tag{16}$$

where only two of the three $(G_t^e,\ G_t^o, \tau_t)$ policy instruments can be exogenously set. Equation (16) is as in e.g. Baxter and King (1993).[10]

When we solve the model, we will choose G_t^e to be exogenously set and then allow either τ_t, or G_t^o, to be the endogenous, residually determined, policy instrument. In other words, to assess the effects of increases in government expenditure on education, we will explore the importance of the financing decisions by studying the use of either changes in the distorting income tax rate, or changes in lump-sum taxes/transfers (the latter serves as a benchmark case). Again this is as in Baxter and King (1993). Further details about policy regimes are contained in subsection 2.5 below.

Finally, note that, when we calibrate the model, the inclusion of G_t^o will make the residually determined value of the income tax rate correspond to the rate which exists in the data. This will allow for a realistic assessment of the trade-offs between increased spending on public goods versus increased distortions due to higher tax rates.

2.4 Stationary decentralized competitive equilibrium

Given the paths of two out of the three policy instruments, say $\{G_t^e, G_t^o\}_{t=0}^{\infty}$, and initial conditions for the state variables, $\left(K_0^h, H_0^h\right)$, a decentralized competitive equilibrium (DCE) is defined to be a sequence of allocations $\{C_t, u_t, e_t, I_t, K_{t+1}, H_{t+1}\}_{t=0}^{\infty}$, prices $\{r_t, w_t\}_{t=0}^{\infty}$ and the tax rate $\{\tau_t\}_{t=0}^{\infty}$ such that (i) households maximize utility; (ii) firms maximize profits; (iii) all markets clear; and (iv) the government budget constraint is satisfied in each time period. Note that market clearing values will be denoted without the superscripts h, f.

Since human capital is the engine of long-run endogenous growth, we transform variables to make them stationary. We first define *per capita* quantities for any variable X as $\overline{X}_t \equiv X_t/N_t$, where $X_t \equiv (Y_t, C_t, I_t, K_t, H_t, G_t^e, G_t^o)$. We next express these quantities as shares of per capita human capital, e.g.

[10] We use a balanced budget. Ignoring public debt is not critical here since changes in lump-sum taxes/transfers are equivalent to debt financing (see e.g. Baxter and King (1993)).

$x_t \equiv \overline{X}_t/\overline{H}_t$. Finally, the gross human capital growth rate is defined as $\gamma_t \equiv \overline{H}_{t+1}/\overline{H}_t$.

Using this notation, substituting out prices $\{r_t, w_t\}_{t=0}^{\infty}$ and substituting for Λ_t and Λ_{t+1} in (8) and (9) respectively, we obtain the following per capita stationary DCE:

$$y_t = A_t \left(k_t\right)^{\alpha_1} \left(1 - e_t\right)^{(1-\alpha_1)} \tag{17a}$$

$$n\gamma_t k_{t+1} - \left(1 - \delta^p\right) k_t + c_t + g_t^e = y_t \tag{17b}$$

$$n\gamma_t = 1 - \delta^h + \left(e_t\right)^{\theta_1} B \left(g_t^e\right)^{\theta_2} \tag{17c}$$

$$\lambda_t = \left(c_t\right)^{-\sigma} \tag{17d}$$

$$\lambda_t = \beta \left(\gamma_t\right)^{-\sigma} E_t \left\{ \lambda_{t+1} \left[1 - \delta^p + \frac{\alpha_1 \left(1 - \tau_{t+1}\right) y_{t+1}}{k_{t+1}} \right] \right\} \tag{17e}$$

$$\psi_t = \left[\frac{\left(c_t\right)^{-\sigma} \left(1 - \alpha_1\right) \left(1 - \tau_t\right) y_t}{\left(1 - e_t\right) \theta_1 \left(e_t\right)^{\theta_1 - 1} B \left(g_t^e\right)^{\theta_2}} \right] \tag{17f}$$

$$\begin{aligned} \psi_t \;=\; & \beta E_t \{ \left(\gamma_t c_{t+1}\right)^{-\sigma} \left(1 - \alpha_1\right) \left(1 - \tau_{t+1}\right) y_{t+1} + \beta \left(\gamma_t\right)^{-\sigma} \times \\ & \psi_{t+1} \left[1 - \delta^h + \theta_1 \left(e_{t+1}\right)^{\theta_1} B \left(g_{t+1}^e\right)^{\theta_2} \right] \} \end{aligned} \tag{17g}$$

$$g_t^e + g_t^o = \tau_t y_t \tag{17h}$$

where λ_t and ψ_t are the transformed shadow prices associated with (3) and (5) respectively in the household's problem.[11]

Therefore, the stationary DCE is summarized by the above system of eight equations in the paths of the following eight variables: (γ_t, y_t, c_t, e_t, k_{t+1}, λ_t, ψ_t, τ_t) given the paths of the exogenously set stationary spending flows, $\{g_t^e, g_t^o\}_{t=0}^{\infty}$ whose motion is defined in the next subsection.

2.5 Processes for fiscal instruments and technology

We next specify the processes governing the evolution of fiscal (spending-tax) policy instruments. This first requires that each government spending item, which has already been expressed as share of $\overline{H}_t$, be rewritten equivalently as a share of output:

$$g_t^j \equiv \widetilde{g}_t^j y_t \tag{18}$$

where $[j = e, o]$, and $\widetilde{g}_t^j \equiv G_t^j / Y_t$.

[11]Note that $\lambda_t = \Lambda_t / \overline{H}_t^{-\sigma}$ and $\psi_t = \Psi_t / \overline{H}_t^{-\sigma}$ where h-superscripts are omitted in a symmetric equilibrium.

We assume that public education expenditure as share of output, $\widetilde{g}_t^e$, follows an AR(1) process:

$$\widetilde{g}_t^e = \left(\widetilde{g}^e\right)^{1-\rho^e} \left(\widetilde{g}_{t-1}^e\right)^{\rho^e} e^{\varepsilon_t^e} \tag{19}$$

where $\widetilde{g}^e$ is a constant specific below, $0 < \rho^e < 1$ is an autoregressive parameter and ε_t^e an *iid* random shock to public education expenditure with a zero mean and constant standard deviation, σ_e. Thus, the innovations ε_t^e represent discretionary education spending changes.

We will call Regime A the benchmark case in which the lump-sum tax /transfer, $\widetilde{g}_t^o$, is the residual policy instrument, while $\widetilde{g}_t^e$ evolves as in equation (19) and τ_t is fixed at a constant positive rate (specified below when we calibrate the model). We will call Regime B the more interesting case in which the income tax rate, τ_t, is the residual policy instrument, while $\widetilde{g}_t^e$ evolves again as in equation (19) and $\widetilde{g}_t^o$ is fixed at a constant share (again specified below when we calibrate the model).

In addition, we will also study the case, called Regime C, in which all types of government spending rise together (see also e.g. Blankenau and Simpson (2004), p. 588). Thus, in our setup, increases in $\widetilde{g}_t^e$ are accompanied by proportional increases in $\widetilde{g}_t^o$. This is like in Regime B, in the sense that τ_t is residually determined and $\widetilde{g}_t^e$ follows equation (19), but now the composition of public expenditure remains constant when government spending as a share of output changes. Formally, in comparison to Regime B, $\widetilde{g}_t^o = \left(\frac{g^o}{g^e}\right)\widetilde{g}_t^e$. The interest is now in the effects of the overall size of the public sector.

Finally, we assume that the stationary stochastic process determining A_t follows an AR(1) process:

$$A_t = A^{(1-\rho^a)} A_{t-1}^{\rho^a} e^{\varepsilon_t^a} \tag{20}$$

where $A > 0$ is a constant, $0 < \rho^a < 1$ is the autoregressive parameter and $\varepsilon_t^a \sim iid(0, \sigma_a^2)$ are the random shocks to productivity.

3 Data, Calibration and Model Fit

3.1 Data

To calibrate the model, we require data for the endogenous variables as shares of human capital. Thus it is important to obtain a measure of human capital that is comparable to monetary valued quantities such as consumption, income, physical capital and government spending. To obtain this, we use data from Jorgenson and Fraumeni (1989, 1992a,b) on human and physical capital.[12] These measures are reported in billions of constant 1982 dollars for 1949-1984.

The basic idea used in the construction of this dataset is that the output of the education sector is considered as investment in human capital. In this context, Jorgenson and Fraumeni (1992a) note: "investment in human beings, like investment in tangible form of capital such as buildings and industrial equipment, generates a stream of future benefits. Education is regarded as an investment in human capital, since benefits accrue to an educated individual over a lifetime of activities". Jorgenson and Fraumeni (1989) also note that "in order to construct comparable measures of investment in human and nonhuman capital, we define human capital in terms of lifetime labor incomes for all individuals in the US population. Lifetime labor incomes correspond to asset values for investment goods used in accounting for physical or nonhuman capital".

The additional (annual) data required for calibration and model evaluation are obtained from the following sources: (i) Bureau of Economic Analysis (NIPA accounts); (ii) OECD (Economic Outlook database); (iii) US Department of Labor, Bureau of Labor Statistics (BLS); and (iv) ECFIN Effective Average Tax Base (see Martinez-Mongay, 2000).

3.2 Calibration

The numeric values for the model's parameters are reported in Table 1. To calibrate the model, we work as follows. We set the value of $(1-\alpha_1)$ equal to labor's share in income (i.e. 0.578) using compensation of employees data from the OECD Economic Outlook. This figure is similar to others used in the literature, see e.g. Lansing (1998). Given labour's share, capital's share, α_1, is then determined residually.

The discount rate, $1/\beta$ is equal to 1 plus the *ex-post* real interest rate, where the interest rate data is from the OECD Economic Outlook. This implies a value 0.964 for β. Again this figure is similar to other US studies, see e.g. King and Rebelo (1999), Lansing (1998) and Perli and Sakellaris (1998). The population gross growth rate n is set equal to the post war labor force growth rate, 1.016, obtained by using data from Bureau of Labor Statistics. The depreciation rates for physical, δ^p, and human capital, δ^h, are calculated by Jorgenson and Fraumeni to be, on average, 0.049 and 0.0178, respectively.

[12] As said above, generally empirical studies use measures of school enrolment ratios or years of schooling as general proxies of labor quality or human capital. However, in our setup, these proxies are measures of the input to the production function of human capital (time spent on education) and not of the output of this activity, new human capital.

We also use a value for the intertemporal elasticity of consumption $(1/\sigma)$ that is common in the DSGE literature (i.e. $\sigma = 2$).

Table 1: Parameter Values (base calibration)

parameter	value	definition
$A > 0$	0.065	technological progress in goods production
$B > 0$	0.637	technological progress in human capital production
$0 < \alpha_1 < 1$	0.422	productivity of private capital
$0 < 1 - \alpha_1 < 1$	0.578	productivity of effective labor
$0 < \beta < 1$	0.964	rate of time preference
$n \geq 1$	1.016	population growth
$0 \leq \delta^p \leq 1$	0.049	depreciation rate on private capital
$0 \leq \delta^h \leq 1$	0.018	depreciation rate on human capital
$0 < \widetilde{g}^e < 1$	0.053	public education spending share of output
$0 < \widetilde{g}^o < 1$	0.157	other public investment spending share of output
$\sigma > 1$	2.000	$1/\sigma$ is the intertemporal elasticity of consumption
$0 \leq \theta_1 \leq 1$	0.600	productivity of household human capital
$0 \leq 1 - \theta_1 \leq 1$	0.400	productivity of aggregate human capital
$0 \leq \theta_2 \leq 1$	0.245	productivity of public education spending
$0 < \tau < 1$	0.210	effective direct tax rate
$0 < \rho^e < 1$	0.442	AR(1) parameter public education spending
$0 < \rho^a < 1$	0.933	AR(1) parameter technology
$\sigma_e > 0$	0.019	std. dev. of public education spending innovations
$\sigma_a > 0$	0.010	std. dev. of technology innovations

We also require constants for government education and other government spending as shares of output. The education spending ratio is set at the data average using NIPA data to $\widetilde{g}^e = 0.053$. We set other government spending, G_t^o, in the government budget equation (16) so that the long-run solution for the tax rate gives 0.21. This value corresponds to the effective income tax reported in the ECFIN dataset.[13] This implies $\widetilde{g}^o = 0.157$. It is important to point out that, given the above data averages, all three policy regimes defined in subsection 2.5 , imply the same long-run solution.

We next move on to the parameters θ_1, θ_2 and B in the production function of human capital and A in the production function of goods. Note that the expression $\left(e_t^h H_t^h\right)^{\theta_1} \left(\overline{H}_t\right)^{1-\theta_1} B \left(g_t^e\right)^{\theta_2}$ in equation (5) is essentially the production function for the creation of new human capital, or what Jorgenson and Fraumeni (1992a, b) call investment in human capital. Model

[13] We calculate this as the weighted average of the effective tax rates on (gross) capital income and labor income, where the weights are capital's and labor's shares in income.

consistent values for the scale parameters A and B are obtained by solving equations (5) and (12) using data averages and long-run values for the variables y, k, e, γ and g^e, as well as the calibrated parameters α_1, θ_1, θ_2, n, and δ^h.[14] Given the functions for the calibration of A and B, we calibrate θ_1 and θ_2 so that we obtain an economically meaningful and data-consistent long-run solution. This can be obtained by setting a value of $(1-\theta_1) = 0.4$ for the human capital externality. Conditional on this value, the productivity of individual human capital is $\theta_1 = 0.6$, and we then calibrate $\theta_2 = 0.245$ to ensure that the long-run gross growth rate of output per labour input is equal to 1.02. Note that a value of the gross growth rate of 1.02 is the USA per labour input growth rate for 1949-1984 using GDP data from the NIPA accounts and labour data from Bureau of Labor Statistics. The long-run solution implied by this calibration is reported in Appendix 6.1.

Regarding the calibrated values of θ_1 and θ_2, it is important to report the following. For higher externalities, the growth rate becomes too low, irrespective of the size of θ_2. This happens because, with very high externalities, there are free riding problems in the creation of human capital. On the other hand, for low externalities, the implied share of time allocated to education (e) in the long-run becomes unrealistically large (around 0.5). By contrast, our calibrated values $(1-\theta_1) = 0.4$ and $\theta_2 = 0.245$ guarantee a growth rate consistent with the data average (2%) and, at the same time, imply that agents in the long-run will spend about a third of their non-leisure time in acquiring human capital and two thirds to work.[15]

To accurately gauge the persistence of the public spending shocks, we estimate the $AR(1)$ relation given by (20) using US NIPA data, where the log deviations are defined relative to a Hodrick-Prescott trend. The estimated values for ρ^e is 0.442 and is significant at less than the 1% level of significance. The standard deviation of this process, σ_e, is 0.019. Using a production function and time period similar to ours, Lansing (1998) provides estimates for TFP. Hence we use his parameters for the stationary TFP process in (20), e.g. $\rho = 0.933$ and $\sigma_a = 0.01$.

[14] For this exercise, we obtain model consistent y. In particular, y is obtained from equation (17b), using NIPA data for g^e and c. As a dataset for the share of time individuals spend on education as opposed to working is not available, we obtain a proxy for e to calibrate B. This is achieved by assuming that e is the ratio of 16 years spent on average on education over 62 minus 6 years available on average for work or education.

[15] As said above, there are no data on the fraction of time agents allocate to increasing human capital as opposed to working. Although there are data on average years of schooling, they can only provide a lower bound on the actual time allocated to learning, as agents take time off work as well for vocational training, to attend seminars, etc. Looking at data for schooling only, our long run solution is a bit higher than the current situation. However, there is a clear trend in schooling data for more years of schooling.

To sum up, this model economy for the post-war USA is consistent with externalities in private human accumulation and productive public education expenditure. Note that the output effect of human capital externality in our economy is lower than in Lucas (1988). Lucas supports the same value but, since his externality is modeled as a direct argument in the goods production function, its effect on output produced is much higher. The associated value of the productivity of public education expenditure, $\theta_2 = 0.245$, is also within the range assumed in the related literature (see e.g. Blankenau (2005) p. 501). We finally note that the possible benefits from human capital and public education can be broader. In other words, we can think of human capital not only as the skills embodied in a worker, but also as ideas, knowledge and social behavior, so that there can be external effects in addition to know-how and learning-by-doing. Also, public education can reduce criminal activity, increase social cohesion, improve incentives, as well as produce innovation.[16]

3.3 Model fit

Before moving on to policy analysis, we first evaluate our model to assess how close the model generated second moments are to those in the actual data (see e.g. King and Rebelo (1999)). To this end, we first compare model generated autocorrelation (ACFs) and cross-correlation functions (CCFs) with those produced by the data. Then, following the usual model evaluation strategy employed in the RBC literature, we compare the relative volatilities of the main variables with respect to output to those of the actual data.

3.3.1 ACFs and CCFs

Given that our model's system matrix is (10x10), it is not practical to report all the ACF and CCF comparisons. Besides the size issue, in some cases, comparisons with the actual data is not possible since the actual data does not exist, e.g. education time.[17] Therefore, to focus the exercise, we report the comparisons for all ACFs where actual data exist and CCFs which are critical for the welfare calculations. For example, the welfare calculations

[16] To check that our calibration does not produce a multiplicity of solutions for any one parameter configuration, we examine the effects on the steady-state growth rate from a range of values for every parameter in the model (see Appendix Figure 1). These ranges cover the values found in the relevant calibration and estimation literatures. Finally note that in all of the analysis which follows, we use the values calibrated in Table 1 to obtain the second-order approximation to the stationary equilibrium as given by equations ($17a-20$).

[17] We would be happy to supply the results not reported on request.

reported in the Appendix show that lifetime utility depends on the steady-state values of c_t and γ_t, but also on the cyclical paths of $\widehat{c}_t$ and $\widehat{\gamma}_t$ around their steady-state values, c and γ.[18] It is therefore also important, for our subsequent policy analysis, that the model generates series for c_t and γ_t whose dynamic paths and co-movements resemble those in the actual data.

To calculate these correlations, we use policy regime B since the results are quantitatively indistinguishable for the other two regimes. To establish 95% confidence intervals for the model moments (see e.g. Canova (2007) ch. 7), we simulate the model 1000 times and present the sample average of the ACFs and CCFs along with upper and lower bounds in Figure 1. We then calculate the ACFs and CCFs from the actual data and plot them in the same graphs, to check whether the actual data correlations are within the bands predicted by the model. As Canova (2007) suggests, we can interpret this as a measure of model fit.[19]

[Figure 1 about here]

Figure 1, subplot (1,2) - where (1,2) refers to row and column numbers respectively - plots the ACF for c, subplot (3,1) the ACF for γ, while subplots (3,2) and (3,3) the CCFs for (c, γ) and (γ, c) respectively. The results show that the correlations of the actual data are similar to those predicted by the model and, with the exception of the correlation of c with the second lag of γ, the distance between the data and the model correlations is not statistically significant. In addition, the ACFs for the actual series for the other variables are also consistent with the model predictions. Overall, then, the dynamic properties of the model generated series appear to resemble those of the actual data for the American economy.

3.3.2 Relative volatilities

As contrasting relative volatilities is a standard exercise in the RBC literature, we also evaluate the distance between the model-actual relative volatilities implied by our model, to the distance between the model-actual relative volatilities implied by the prototype RBC model as discussed in King and Rebelo (1999). We hence focus on the main variables of interest in this literature. This helps to contextualize the successes and failures of our model relative to those of the RBC literature. We start by calculating the standard deviation of variable j relative to that of y. To evaluate the statistical

[18] In a linearized setup these deviations sum to zero. However in our second-order setup these are a non-zero constant (see, e.g. Schmitt-Grohé and Uribe (2004)).

[19] Also note that, following the literature, both the actual and the model generated series are logged and HP filtered before obtaining the second moments of interest.

significance of the distance between the model and the actual relative standard deviations, we again calculate a 95% confidence interval for the relative standard deviations of the model generated series using 1000 simulations. Results are presented in Table 2.

Columns (1) and (2) in Table 2 repeat the data and model relative standard deviations for the USA economy in King and Rebelo (1999).[20] Columns (3)-(6) report the data and model relative standard deviations for our model under regime B (again since the other two regimes produce similar results).[21] The results suggest that the main qualitative characteristics for the model generated and the actual relative volatilities are in general similar in our model. In particular, consumption (c) is less volatile than output, investment (i) is about three times more volatile, the wage rate (w) is less volatile and the interest rate (r) is less volatile.

The main failure is with respect to data for hours worked (N), which seems to be more or less as volatile as output in the data, while our model predicts less than half a standard deviation for the labor input to the production function (u) relative to that of output. This is a common failure of the basic RBC model (see columns (1) and (2)) and actually not a precise comparison to our model, as we do not have an endogenous leisure choice, in addition to the work effort/education time choice, as explained above. Despite that, the relative volatility of the labor input in our model is not qualitatively different from the relative volatility of the prototype RBC model with endogenous leisure choice.[22]

Finally, to compare our model fit regarding relative volatilities to that of the prototype RBC model, we look at the ratio of model to data relative standard deviations for both the King and Rebelo (1999) model and our model. Our model results are clearly better for consumption and the interest rate, while they are essentially the same for investment and the wage rate. We do worse regarding the labor input relative volatility, as discussed above.

Overall, the results from this section suggest that the statistical properties of the model generated data are similar enough to those of the actual data, so that we can have some confidence in using the model to address the policy

[20] The numbers are obtained from Tables 1 and 3 in their paper.

[21] We re-calculate the relative standard deviations for the USA data and do not simply use those reported in King and Rebelo (1999), as they define variables in per capita terms, while in our model, the stationary variables we solve for are expressed as shares of human capital. In addition, King and Rebelo use quarterly data for 1947 to 1996, while we use annual for 1949 to 1984.

[22] The reason is that, as shown in the impulse response analysis below, agents react, on average, to TFP and policy shocks by changing their time allocation. In any case, as pointed out above, we do not have actual data on education time to compare the volatility of e in our model to that of actual data.

questions we have posed in this paper.

Table 2: Business cycle statistics

σ_j/σ_y : KR (1999)			σ_j/σ_y			
	data	model	data	$-2\times(se)$	model	$+2\times(se)$
j	(1)	(2)	(3)	(4)	(5)	(6)
C	0.74	0.44	0.58	0.57	0.61	0.66
I	2.93	2.95	3.03	2.19	2.51	2.83
N	0.99	0.48	0.89	0.22	0.36	0.50
w	0.38	0.54	0.60	0.80	0.88	0.96
r	0.16	0.04	0.45	0.13	0.14	0.15

Model to data relative standard deviation

	(2)/(1)	(4)/(3)	(5)/(3)	(6)/(3)
C	0.60	0.98	1.05	1.14
I	1.01	0.72	0.83	0.93
N	0.49	0.25	0.40	0.56
w	1.42	1.33	1.47	1.60
r	0.25	0.29	0.31	0.33

4 Results

Using the solution of the second-order approximation of the model around its deterministic long-run (see Appendix 6.1 for the long-run solution),[23] we first examine, in subsection 4.1, the effects on the long-run growth rate and expected lifetime utility from varying public education spending shares. This is captured by changes in $\widetilde{g}^e$ in equation (19) and can be interpreted as permanent changes in public education expenditure as share of output.

The above analysis is carried out under all three Regimes, A, B and C, where the point of departure is the US economy as summarized in Table 1. Recall that regime A is the benchmark case where the government relies on changes in lump-sum taxes/transfers to finance changes in public education spending (with the income tax rate remaining at its calibrated value of $\tau = 0.21$). In Regime B, the same spending changes are financed by changes in the income tax rate (with lump-sum taxes/transfers remaining at their calibrated value of $\widetilde{g}^o = 0.157$). In Regime C, the government is increasing both $\widetilde{g}^o_t$ and $\widetilde{g}^e$ by the same proportion, so that the overall size of the government is increasing and is being financed by changes in the income tax rate.

[23] We have written extensive Matlab code to conduct this analysis and make use of the Matlab functions made available by Schmitt-Grohé and Uribe (2004) to solve and simulate the second-order approximation of the model.

In subsection 4.2, we examine the effects of the same changes in $\widetilde{g}^e$ on some other key endogenous variables. In turn, in subsection 4.3, we examine the effects on expected lifetime utility from varying degrees of uncertainty over public education spending and total factor productivity. This is captured by changes in σ_e and σ_a in equations (19) and (20) respectively. Finally, in subsection 4.4, we examine the transition paths to the long-run when temporary shocks are applied to the innovations of the exogenous processes driving public education spending and total factor productivity.

4.1 Effects of public education spending on growth and welfare

As is well known, the relationship between government size and economic growth is not expected to be monotonic. On one hand, governments provide growth promoting public goods and services and on the other, this provision requires taxes and distorts incentives. There is thus a trade-off being reflected in a Laffer curve between government size and economic growth (see e.g. Barro (1990)).[24] A similar pattern is expected to describe the relationship between government size and welfare.

In our setup, it is useful to have a handle on the magnitudes associated with the growth maximizing size of public education spending and whether this is also welfare maximizing. We will therefore first calculate the long-run growth rate for a range of shares of public education spending in GDP, and then do the same with welfare. We do so under each policy regime. These calculations will allow us to find the shares that yield the maximum long-run growth rate and the maximum welfare.

4.1.1 Measures of long-run growth and expected lifetime utility

First, consider long-run growth. This is measured by the balanced growth rate γ, which is the common constant rate at which all quantities grow in the long-run. In Figure 2, subplot (1,1), where (1,1) refers to row and column numbers respectively, we derive the Laffer curve for the steady-state growth rate implied by our model under policy regime A and in subplots (1,2) and (1,3) we derive the Laffer curves under policy regimes B and C.

[Figure 2 about here]

[24] Building upon Barro's model, there are rich and growing theoretical and empirical literatures on the relationship between fiscal policy and economic growth. See e.g. Malley *et al.* (2007), who also review these literatures.

Next, consider welfare. This is defined as the conditional expectation of the discounted sum of lifetime utility (see eq. (1)). To this end, we first undertake a second-order approximation of the within-period utility function (see eq. (2)) around the non-stochastic steady-state and then take the discounted "infinite" sum of approximate within-period utility functions (see Appendix 6.2). We do so for varying shares of public education spending ($\widetilde{g}^e$) using the solution(s) of the second-order approximation to the stationary equilibrium as given by equations ($17a - 20$). Note that in comparison to the related literature (see e.g. Schmitt-Grohé and Uribe (2007)), we work with an endogenous growth model. The resulting welfare curves are shown in subplots (2,1), (2,2) and (2,3) in Figure 2 for policy regimes A, B and C respectively.

In turn, working as in e.g. Lucas (1990), Cooley and Hansen (1992) and Schmitt-Grohé and Uribe (2007), we also compute the welfare gains, or losses, associated with alternative policy regimes by computing the percentage change in private consumption that the individual would require so as to be equally well off between two regimes. This is defined as "ξ" (see Appendix 6.2 for the derivation of ξ in our model). Subplots (3,1), (3,2) and (3,3) show ξ - respectively, the welfare gain/loss from moving from A to B, A to C, and B to C - for a range of values of $\widetilde{g}^e$.

Note that, in general, long-term growth maximizing, and lifetime utility maximizing, policies are not expected to be the same since the latter (namely, lifetime utility) also includes compositional and stochastic steady-state effects.

4.1.2 Results for long-run growth and expected lifetime utility

Subplot (1,1) in Figure 2, describing the benchmark case (Regime A) in which changes in public education spending are financed by changes in lump-sum taxes/transfers, does not give a Laffer curve, i.e. the benefits always outweigh the costs at least in the range of parameter values we use. This is not surprising given the assumed non-distorting way of financing growth-enhancing government spending like public education. In policy regime B, in subplot (1,2), there is a trade-off and hence a Laffer curve. The growth maximizing public education share is around 50% of GDP, which implies an overall government size of around 65% of GDP, with the associated gross growth rate being 1.043%. For policy regime C, in subplot (1,3), the growth maximizing $\widetilde{g}^e$ is around 15% of GDP, with the associated gross growth rate being 1.026%.

While the above growth Laffer curves show that there is scope for higher public education spending relative to the data average, maximization of long-run growth cannot take into account compositional effects as well as business cycle fluctuations. Hence we also study expected lifetime utility. Subplots (2,1), (2,2) and (2,3) suggest that the welfare maximizing $\widetilde{g}^e$ share is much smaller than the growth maximizing one, and practically is around the data average which is 5.3%. In particular, under regime A (subplot (2,1)), the welfare maximizing $\widetilde{g}^e$ is around 6%, under policy regime B (subplot (2,2)), the welfare maximizing $\widetilde{g}^e$ is approximately 4.5% and in regime C the welfare maximizing $\widetilde{g}^e$ is found to be about 2.5% of GDP. The implied overall welfare maximizing size of the government (or the direct tax rate) is 21%, 19.5% and 10% in Regimes A, B and C respectively.

The main reason that the welfare Laffer curves peak much earlier than the respective growth Laffer curves is that growth, being driven by productive government spending, has also adverse compositional effects. In particular, in all three regimes A, B and C, although a higher government size enhances growth by allocating more social resources towards public education, there are also crowing-out effects on private investment and private consumption that work in opposite directions and after a critical value reduce net welfare. This happens simply because, when the government increases its spending, it purchases a part of the private output.

In regimes B and C, there is an additional adverse effect from higher government spending: there is an extra fall in private consumption because the required increase in distorting tax rates reduces post-tax income. Comparing regimes B and C, the adverse effects due to higher taxation, are worse in C as expected. Therefore, the welfare maximizing $\widetilde{g}^e$ is highest in regime A and lowest in regime C, with B in between. These effects are confirmed when we present the behavior of other stationary variables and impulse response functions below.

Finally, consider welfare comparisons as illustrated in the last row of Figure 2. Subplot (3,1) suggests that for sizes of public education spending above the data average (5.3%), regime A welfare dominates regime B, since in regime B increases in public spending necessitate higher distorting taxes. For instance, for $\widetilde{g}^e = 6\%$, the percentage gain in private consumption is around 0.7%, while for $\widetilde{g}^e = 10\%$, the percentage gain in private consumption is around 3.2%. The same subplot implies that for sizes of public education spending below the data average, regime B welfare dominates regime A for symmetrically opposite reasons; namely, in regime B decreases in public spending allow lower distorting taxes. For instance, for $\widetilde{g}^e = 4.5\%$, the percentage gain in private consumption from switching from A to B is around 0.2%.

Subplots (3,2) and (3,3) respectively imply that regime A welfare dominates regime C and regime B welfare dominates regime C when public education spending is above the data average, and symmetrically opposite when public education spending is below the data average. The welfare gains are much larger when the comparison is made with respect to regime C. For instance, for $\widetilde{g}^e = 6\%$, the percentage gain in private consumption from moving from regime C to A is around 1.9%, while the percentage gain in private consumption from moving from regime C to B is around 1.2%. Hence, scenario C, where all types of government spending rise by the same proportion and are financed by higher income taxes, is clearly the worst regime.

Therefore, the policy implication is that if the government aims to raise the share of public education spending above the data average, in terms of lifetime utility, regime A welfare dominates B which in turn dominates regime C. And symmetrically opposite: if the government aims to decrease the share of public education spending below the data average, in terms of lifetime utility, regime C welfare dominates B which in turn dominates regime A.

4.2 Effects on other endogenous variables

To further examine the importance of the fiscal policy regimes for the economy, we briefly discuss the long-run effects of changes in the $\widetilde{g}^e$ share on some other key variables of the model under the three different regimes. We focus on education time (e), private consumption (c), output (y), the marginal values of consumption, (λ) and of education, (ψ) and the tax rate (τ). These are shown in Figure 3. Since the three regimes assume different financing policies, their effects on the tax rate are different. For the other variables, the effects are, in general, qualitatively similar in all three regimes (with the exception of ψ, as will be discussed). In addition, for a range of $\widetilde{g}^e$ between the welfare maximizing shares under the three regimes, i.e. from 2.5% to 6%, which practically covers the empirically relevant $\widetilde{g}^e$ shares, the effects are quantitatively close as well.

[Figure 3 about here]

By design, changes in $\widetilde{g}^e$ in regime A have no effect on the tax rate, while the effect is larger in regime C, as can also be seen in subplot (3.2) in Figure 3. These bigger changes in the tax rate imply, in general, larger effects on the economy in regime C, although for $\widetilde{g}^e$ in the range 2.5% to 6%, these are essentially sizeable for consumption, output and the marginal value of education only. Regarding c and y, the difference is only quantitative. The difference, however, is qualitative for ψ.

In principle, increases in the size of public education spending imply an increase in the return to investing effort in education, as public education spending augments the increase in human capital associated with individual effort. In other words, the "marginal human capital product of education time" is increased. This tends to increase the marginal value of education. However, the increase in the tax rate, associated with regimes B and C decreases the return to investing effort in education. This tends to decrease the marginal value of education. For regime C, where the increase in the tax rate is high, this latter effect dominates so that ψ is reduced when $\widetilde{g}^e$ increases. The opposite is true for decreases in $\widetilde{g}^e$.

Regarding the effects of higher public education spending on the other (than ψ) endogenous variables presented in Figure 3, the most interesting results are the fall in c and y. These reflect crowding out effects, and they are behind the negative effect of $\widetilde{g}^e$ on welfare, providing thus the main reason that the welfare Laffer curves peak much earlier than the growth Laffer curves, as explained in the previous subsection.

Regarding e, note that the increase in the provision of the public education good creates incentives for private agents to free ride on public education services, and hence reduce their effort in education. Finally, λ - or, in equilibrium, the implicit return to investment - increases with $\widetilde{g}^e$. As increases in public education increase human capital, this will increase as well the return to investment in physical capital, as human and physical capital are complements in our production process.

4.3 Effects of uncertainty

This subsection examines the effects of uncertainty on welfare differences between policy regimes. Recall that there are two sources of uncertainty in our model: the shock to government education spending whose standard deviation is σ_e in (19), and the shock to TFP whose standard deviation is σ_a in (20). We examine the effects of increases in σ_e while keeping σ_a at its data estimate, and vice versa for the TFP shock. We present results for $\widetilde{g}^e = 4.5\%$, which is the welfare maximizing share under regime B, and $\widetilde{g}^e = 6\%$, which is the welfare maximizing share under regime A. Recall that 4.5% is below the data average, while 6% is above the data average.

The results are shown in Figure 4a. Increases in uncertainty of any type do not change the qualitative results obtained above regarding which regime is preferred. In other words, regime A welfare dominates regime B for a $\widetilde{g}^e$ share of 6%, as ξ remains positive (and vice versa for $\widetilde{g}^e$ at 4.5%). Also, regimes A and B welfare dominate C for a $\widetilde{g}^e$ share of 6% as the relevant values of ξ again remain positive (and vice versa for $\widetilde{g}^e$ at 4.5%). However, increases in uncertainty have quantitative effects: they affect the magnitude of ξ, i.e. how much one regime is preferred over the other. In general, increases in σ_e or σ_a increase the difference of regime C from regimes A and B, while they reduce the difference between A and B. In particular, as σ_e or σ_a rise, the inferiority (resp. superiority) of regime C gets bigger when $\widetilde{g}^e = 6\%$ (resp. $\widetilde{g}^e = 4.5\%$). On the other hand, as σ_e or σ_a rise, the inferiority (resp. superiority) of B relative to A gets smaller when $\widetilde{g}^e = 6\%$ (resp. $\widetilde{g}^e = 4.5\%$).

[Figure 4a about here]

In order to understand these results, we also present Figure 4b, which shows the effects of higher σ_e and σ_a on the unconditional means, volatilities and correlation of $\widehat{c}_t$ and $\widehat{\gamma}_t$. These statistics are reported for a $\widetilde{g}^e$ share of 6% only, since they are practically the same for the 4.5% share. The welfare calculations shown in the Appendix indicate that what matters for welfare, in addition to the steady-state values of c_t and γ_t, is the sum of $\widehat{c}_t$ and $\widehat{\gamma}_t$, which enter positively, the sum of squared $\widehat{c}_t$ and $\widehat{\gamma}_t$, which enter negatively, and their cross-products, which enter negatively. In other words, welfare increases when $\widehat{c}_t$ and $\widehat{\gamma}_t$ get higher on average, less volatile and less correlated.

The rest of this subsection interprets results. We start with the effects of higher σ_e focusing on the case in which $\widetilde{g}^e = 6\%$. In all regimes, a higher σ_e increases the average value of $\widehat{c}_t$, decreases the average value of $\widehat{\gamma}_t$, increases volatility, and decreases the correlation between $\widehat{c}_t$ and $\widehat{\gamma}_t$. The average value of $\widehat{\gamma}_t$ decreases because the return to human capital becomes more uncertain, given the increase in the volatility of government spending shocks, so that households put less effort in education and thus less human capital is produced. As the return to physical capital follows the return to human capital, since the two forms of capital are complements in the goods production function, investment is also reduced and consumption is increased; hence, the increase in the mean of $\widehat{c}_t$. Bigger shocks produce bigger reactions so that, naturally, the volatility of $\widehat{c}_t$ and $\widehat{\gamma}_t$ increase. Finally, the correlation between $\widehat{c}_t$ and $\widehat{\gamma}_t$ decreases, as they move in opposite directions after a government spending shock (see also the impulse responses below).

The increase in $\widehat{c}_t$ and the fall in the correlation of $\widehat{c}_t$ and $\widehat{\gamma}_t$ tend to increase welfare, but the other effects to reduce it. Actually, the negative effects dominate in all regimes, so that welfare falls when uncertainty rises in all regimes, as expected under risk aversion. These effects are more pronounced in regime C. The reason is that in this regime, shocks to public education spending are propagated and amplified through the economy, as - in this regime - such shocks have a relatively big effect on the tax rate (see again the impulse responses below). Hence, regime C becomes even worse in terms of welfare. This propagation of the public education spending shocks via the tax rate also takes place in regime B relative to A. However, this works in favour of regime B. In the latter, relative to A, there are more gains by the increase in the average value of $\widehat{c}_t$ and the fall in the correlation between $\widehat{c}_t$ and $\widehat{\gamma}_t$, so that regime A gets less attractive.

Overall, the net effect of the above implies that as σ_e increases, regime C gets even worse and the superiority of A over B gets smaller. The latter means that the welfare gains associated with using lump-sum versus distorting taxes to finance public education expenditure decrease.

[Figure 4b about here]

We continue with the effects of higher σ_α focusing again on the case in which $\widetilde{g}^e = 6\%$. A first feature is that, with the exception of the mean of $\widehat{\gamma}_t$, the effects in regime C are less pronounced than in the previous case. The reason is that, in all regimes, this shock does not affect the tax rate, so that the above amplification mechanism does not work.

A second feature is that the average $\widehat{\gamma}_t$ increases as σ_a increases. The volatility of total factor productivity shocks hurts the wage income and hence makes future labor income more uncertain. In order to smooth labor income, optimizing agents react by putting more effort in human capital, hence increasing their own productivity and making future earnings less dependent on total factor productivity. This results in an increase in human capital - hence the increase in $\widehat{\gamma}_t$. However, the increase in $\widehat{\gamma}_t$ is smaller in regime C than in A and B. This happens because - for given σ_α - there is higher volatility in the tax rate under regime C driven by shocks to public education (recall that σ_e remains at its data average during this experiment), so that the future returns on human capital are more volatile and so education effort is discouraged.

A third feature is that the mean of $\widehat{c}_t$ falls. This happens because, given the complementarities between physical and human capital, investment in the former rises in all regimes and therefore the mean of $\widehat{c}_t$ falls.

A fourth feature is that, as total factor productivity shocks affect growth and consumption in the same direction, the correlation between $\widehat{c}_t$ and $\widehat{\gamma}_t$ increases as σ_a rises.

Overall, the net effect is as that of σ_e above: namely, as σ_a increases, regime C gets even worse and the superiority of A over B gets smaller.

4.4 Impulse response functions

We next turn to the dynamic properties of the model and examine the effects of temporary shocks to public education spending and TFP innovations.

4.4.1 Education spending shock

In Figure 5a, we start our analysis by examining the response to a standard deviation shock to $\widetilde{g}_t^e$ innovations (see eq. (19)). We present results for all three regimes. The first subplot (1,1) in Figure 5a presents the response of the education share, while the last subplot (3,3) shows the implication of this shock for the tax rate, given the different fiscal financing assumptions made in each regime. With the exception of the wage rate, the responses of the endogenous variables are qualitatively similar for all three regimes. In general, they are larger for regime C, as this regime implies larger changes in the fiscal size of the government.

The most important thing to note in Figure 5a is that an increase in g_t^e generates a non-monotonic reaction in the growth rate, γ_t (subplot (1,2)). Initially, the positive effects of increased human capital productivity outweigh the disincentives of higher taxation, τ_t (subplot (3,3)) so that education time, e_t (subplot (1,3)) and hence γ_t increase. However, after the initial boom the negative effects dominate, thus the growth rate becomes lower than its steady-state value after four years and the economy converges to the steady-state from below.

Since g_t^e enters the human capital production function directly (see eq. (5)), human capital increases by more than output, consumption and private capital, so that these variables as shares of human capital, i.e. y_t, c_t, k in subplots (2,1), (3,1), (2,2), fall.

[Figure 5a about here]

We then examine the post-tax wage rate, $(1-\tau)w_t$, subplot (2,3) after the education spending shock. The post-tax wage rate decreases for regime C but for the other two regimes it initially increases and then it falls below its steady-state value. This can be understood as follows. The increase in education time, e_t, after the public spending shock, implies a decrease in labor supply and therefore pushes wages upwards (see eq. (14)). On the other hand, the increase in the tax rate, in regimes B and (especially) C tend to reduce $(1-\tau)w_t$. The latter effect dominates in regime C. In regimes A and B, the positive effect from e_t initially dominates, but as e_t decreases over time (see subplot (1,3)), the wage rate falls below the steady-state for these regimes as well.

Finally, we examine the response of the post-tax interest rate,$(1-\tau)$ r_t, subplot (3,2) after the education spending shock. The return to capital r_t initially falls, but subsequently increases. This follows the movement of the ratio y_t/k_t, see eq. (15). Initially, y_t decreases by more than k_t, so that the interest rate, r_t is reduced, but after 3-4 periods y_t is increasing faster than

k_t so that the interest rate converges to its steady-state value from above. The effects for the post-tax interest rate are again pronounced for regime C, as the changes in the tax rate are higher in this regime.

4.4.2 TFP shock

In Figure 5b, we finally examine the responses of the same endogenous variables to a standard deviation shock to TFP innovations, see, eq. (20) (as the results are the same for all policy regimes, we only plot one line in Figure 5b). This will help to contextualize the qualitative and quantitative importance of the education spending shock. The first thing to note is that, in the absence of the trade-off associated with discretionary government education spending discussed above, the responses after the TFP shock are positive for the growth rate, output, consumption and private capital. The second important observation is that the quantitative magnitudes of the reactions are comparable for the standard deviation TFP and education spending shocks.

[Figure 5b about here]

By definition, a positive TFP shock increases the productivity of both effective labour, see w_t in subplot (2,3) and physical capital, see r_t in subplot (3,2) (note that in this exercise, the tax rate, τ_t (subplot (3,3)) does not change). Given the increase in r_t, investment in physical capital increases, as reflected by the increase in k_t (subplot (2,2)). However, as y_t is decreasing faster than k_t, r_t (see eq. (15)) falls below its steady-state value and converges to this from below. Regarding e_t (subplot (1,3)), it initially falls, as households want to work more to receive the higher wage, but the increased return to human capital implies that they also want to educate more, so that after the initial drop, e_t is increased and this helps to sustain the increase in the growth rate for about 15 years.

Since, via the production function for goods (see eq. (12)), the effect of TFP on physical goods production is direct, output increases by more than human capital, so that y_t (subplot (2,1)) increases. As consumption (subplot (3,1)) is a normal good, it follows income by increasing, although by a smaller amount. This is the standard consumption smoothing result.

5 Conclusions

In this paper, we have examined the importance of public education spending, under human capital externalities, for aggregate outcomes. Our main findings include the following:

First, there is evidence of positive externalities from the average stock of human capital in the economy. There is also evidence that public education expenditure augments private human capital accumulation. Without these features, we cannot obtain a data-consistent long-run equilibrium.

Second, the growth and welfare effects of public education expenditure are not monotonic. Specifically, the pattern between public education expenditure and long-run growth, as well as the pattern between public education expenditure and lifetime utility, are Laffer curves with the peak of the curves giving the growth-maximizing and the welfare-maximizing public education expenditure as share of output. Given that higher public education spending crowds-out private consumption, the welfare maximising share is much smaller than the growth maximising one. In fact, we find that the welfare maximizing public education expenditure share is around the data average, which is 5.3% of GDP.

Third, substantial welfare gains are obtainable if increases in public education spending are accompanied by changes in the government tax-spending mix. In particular, welfare gains can be made if the composition of public spending is altered in favour of education spending relative to the other components of total government spending. For instance, departing from the mean of the US economy, and if the public education share rises to, say, 6% of GDP, there are welfare gains that amount to 1.9% of private consumption from reallocating public funds in favor of spending on public education, instead of increasing all types of government expenditure together and thus unavoidably raising distorting taxes.

Fourth, increases in uncertainty (namely, the volatility of the innovations to the processes driving public education spending and total factor productivity) reduces the welfare gains associated with using lump-sum versus distorting taxes to finance public education expenditure.

Fifth, we find that a positive public education spending shock generates a non-monotonic reaction in the growth rate and other key economic variables, so that after an initial boom there is a decrease in the growth rate below its steady-state value. The quantitative effects of discretionary fiscal policy of this form are not trivial as they are similar in magnitude to those of a standard TFP shock.

The general message arising from this study is that public education policy is indeed quantitatively important for macroeconomic outcomes. More research is needed to assess the empirically relevant channels through which various public educations policies shape these outcomes.

References

[1] Azariadis C. and A. Drazen (1990): Threshold externalities in economic development, Quarterly Journal of Economics, 105, 501-526.

[2] Barro R. (1990): Government spending in a simple model of endogenous growth, Journal of Political Economy, 98, S103-S125.

[3] Baxter M. and R. King (1993): Fiscal policy in general equilibrium, American Economic Review, 83: 315-334.

[4] Benhabib J. and R. Perli (1994): Uniqueness and indeterminacy: On the dynamics of endogenous growth, Journal of Economic Theory, 63, 113-142.

[5] Blankenau W. and N. Simpson (2004): Public education expenditures and growth, Journal of Development Economics, 583-605.

[6] Blankenau W. (2005): Public schooling, college education and growth, Journal of Economic Dynamics and Control, 29, 487-507.

[7] Canova F. (2007): Methods for Applied Macroeconomic Research, Princeton University Press, Princeton.

[8] Ciccone A. and G. Peri (2006): Identifying human-capital externalities: theory with applications, Review of Economic Studies, 73, 381-412.

[9] Cooley T. F. and G. D. Hansen (1992): Tax distortions in a neoclassical monetary economy, Journal of Economic Theory, 58, 290-316.

[10] Gong G., A. Greiner and W. Semmler (2004): The Uzawa-Lucas model without scale effects: theory and empirical evidence, Structural Change and Economic Dynamics, 15, 401-420.

[11] Glomm G. and B. Ravikumar (1992): Public versus private investment in human capital: endogenous growth and income inequality, Journal of Political Economy, 100, 818-834.

[12] Heckman J. and P. Klenow (1997): Human capital policy, University of Chicago, *mimeo*.

[13] Jorgenson D. and B. Fraumeni (1989): The accumulation of human and nonhuman capital, 1948-1984, in R.E. Lipsey and H.S. Tice (eds.), The Measurement of Saving, Investment and Wealth, Studies in Income and Wealth, Vol. 52, Chicago, University of Chicago Press.

[14] Jorgenson D. and B. Fraumeni (1992a): The output of the education sector, in Z. Griliches (ed.) Output Measurement in the Services Sector, Studies in Income and Wealth, Vol. 55, Chicago, University of Chicago Press.

[15] Jorgenson D. and B. Fraumeni, (1992b): Investment in education and U.S. economic growth, Scandinavian Journal of Economics, 94, S51-S70.

[16] Judd K. (2000): Is education as good as gold? A portfolio analysis of human capital investment, *mimeo*, revised, May 2000.

[17] King R. and S. Rebelo (1999): Resuscitating real business cycles, in Handbook of Macroeconomics, vol. 1B, edited by J. Taylor and M. Woodford, North Holland.

[18] Lansing K. (1998): Optimal fiscal policy in a business cycle model with public capital, Canadian Journal of Economics, 31: 337-364.

[19] Lucas R. E. (1988): On the mechanics of economic development, Journal of Monetary Economics, 22, 3-42.

[20] Lucas R. E. (1990): Supply-side economics: An analytical review, Oxford Economic Papers, 42, 293-316.

[21] Lucas R. E. (2002): Lectures on Economic Growth, Harvard University Press, Cambridge, Massachusetts and London, England.

[22] Malley J., A. Philippopoulos and U. Woitek (2007): Electoral uncertainty, fiscal policy and macroeconomic fluctuations, Journal of Economic Dynamics and Control, 31, 1051-1080.

[23] Mamuneas, T., P. Kalaitzidakis, A. Savvides and T. Stengos (2001): Measures of human capital and nonlinearities in economic growth, Journal of Economic Growth, 6, 229-254.

[24] Martinez-Mongay C. (2000): ECFIN's effective tax rates. Properties and comparisons with other tax indicators, DG ECFIN Economic Papers, No 146, Brussels: European Commission.

[25] Ortigueira S. (1998): Fiscal policy in an endogenous growth model with human capital accumulation, Journal of Monetary Economics, 42, 323-355.

[26] Perli R. and P. Sakellaris (1998): Human capital formation and business cycle persistence, Journal of Monetary Economics, 42, 67-92.

[27] Romer P. M. (1986): Increasing returns and long-run growth, Journal of Political Economy, 94, 1002-1037.

[28] Schmitt-Grohé S. and M. Uribe (2004): Solving dynamic general equilibrium models using a second-order approximation to the policy function, Journal of Economic Dynamics and Control, 28, 755-775.

[29] Schmitt-Grohé S. and M. Uribe (2007): Optimal simple and implementable monetary and fiscal rules, Journal of Monetary Economics, 54, 1702-1725.

[30] Stokey N. and S. Rebelo (1995): Growth effects of flat-rate taxes, Journal of Political Economy, 105, 519-550.

[31] Su X. (2004): The allocation of public funds in a hierarchical educational system, Journal of Economic Dynamics and Control, 28, 2485-2510.

[32] Tamura R. (1991): Income convergence in an endogenous growth model, Journal of Political Economy, 99, 522-540.

[33] Zhang J. (1996): Optimal public investments in education and endogenous growth, Scandinavian Journal of Economics, 98, 387-404.

6 Appendix

6.1 Steady-state of DCE

In the absence of stochastic shocks to A_t and g_t^e for all t, the economy converges to its steady-state in which the stationary variables defined in subsection 2.4 are constant (i.e. for any x_t, x denotes its long-run value). Since our model's steady-state does not have an analytic solution, its numeric solution implied by the parameters values in Table 1 is reported in Appendix Table 1. With the exception of the steady-state growth rate, γ, and the share of time allocated to education, e, all endogenous variables are reported as shares of both $\overline{H}_t$ and $\overline{Y}_t$. Given that the normalization with respect to $\overline{H}_t$ is less familiar to most readers, the shares with respect to $\overline{Y}_t$ are included to help contextualize the results relative to others reported in the literature.

Appendix Table 1: Steady-state			
variable	Shares of $\overline{H}_t$	variable	Shares of $\overline{Y}_t$
γ	1.02	$n.a.$	$n.a.$
e	0.3346	$n.a.$	$n.a$
$y = \frac{\overline{Y}}{\overline{H}}$	0.0118	$\frac{y}{y}$	1.0000
$c = \frac{\overline{C}}{\overline{H}}$	0.0086	$\frac{c}{y}$	0.7256
$k^p = \frac{\overline{K}^p}{\overline{H}}$	0.0308	$\frac{k^p}{y}$	2.600

[Appendix Figure 1 about here]

6.2 Welfare

6.2.1 Within-period utility function

Using the notation set out in the paper, first consider the per capita representation of the instantaneous CRRA-utility function given by (2)

$$\overline{U}_t = \frac{\overline{C}_t^{1-\sigma}}{1-\sigma} \qquad (A1)$$

or using our notation for stationary variables

$$\overline{U}_t = \frac{\left(c_t \overline{H}_t\right)^{1-\sigma}}{1-\sigma} \qquad (A2)$$

where $c_t \equiv \left(\frac{\overline{C}_t}{\overline{H}_t}\right)$, is stationary consumption and $\overline{H}_t$ is the beginning-of-period capital stock. From the definition of the growth rate, $\gamma_t \equiv \overline{H}_{t+1}/\overline{H}_t$ it follows that for $t \geq 1$

$$\overline{H}_t = \overline{H}_0 \left(\prod_{s=0}^{t-1} \gamma_s\right) \qquad (A3)$$

where $\overline{H}_0$ is given from initial conditions.

Substituting $(A3)$ into $(A2)$ gives

$$\overline{U}_t = \frac{\left(\overline{H}_0 c_t \left(\prod_{s=0}^{t-1} \gamma_s\right)\right)^{1-\sigma}}{1-\sigma} \text{ for } t \geq 1 \qquad (A4.a)$$

$$\overline{U}_0 = \frac{\left(\overline{H}_0 c_0\right)^{1-\sigma}}{1-\sigma} \text{ for } t = 0. \qquad (A4.b)$$

6.2.2 Utility at the steady-state

We next define the long-run as the state without stochastic shocks which implies that stationary variables are constant. Thus using $(A4.a, b)$, utility at the steady-state can be written as

$$\overline{U}_t^* = \frac{\left(\overline{H}_0 c\gamma^t\right)^{1-\sigma}}{1-\sigma}. \tag{A5}$$

where the $*$ superscript denotes steady-state per capita utility. In the steady-state, non-stationary $\overline{C}_t$ grows at the constant rate γ, which in turn implies for $\sigma, \gamma > 1$ that the growth of $\overline{U}_t^*$ is constant and less than unity.

6.2.3 Approximate within-period utility

Define a variable $z_t = c_t \left(\prod_{s=0}^{t-1} \gamma_s\right)$, so that utility is given by, $\overline{U}_t = \frac{(z_t)^{1-\sigma}}{1-\sigma}$ which can be approximated as,

$$\overline{U}_t^s \simeq \overline{U}_t + \left[\overline{z}^{1-\sigma}\right] \widehat{z}_t + \frac{1}{2}\left\{\left[(1-\sigma)\overline{z}^{1-\sigma}\right] \widehat{z}_t^2\right\}. \tag{A6}$$

Then taking the second-order approximation $\widehat{z}_t = \widehat{c}_t + \sum_{s=0}^{t-1} \widehat{\gamma}_s$ (there are only linear terms as the definition of z is exactly log-linear) we obtain,

$$\begin{aligned} \overline{U}_t^s \quad &\simeq \quad \overline{U}_t + \left[\left(\overline{H}_0 \overline{c}\overline{\gamma}^t\right)^{1-\sigma}\right] (\widehat{c}_t + \sum_{s=0}^{t-1} \widehat{\gamma}_s) && (A7a) \\ &+ \left\{\frac{1}{2}\left[(1-\sigma)\left(\overline{H}_0 \overline{c}\overline{\gamma}^t\right)^{1-\sigma}\right] (\widehat{c}_t + \sum_{s=0}^{t-1} \widehat{\gamma}_s)^2\right\} \end{aligned}$$

At time $t = 0$ the equivalent expression is,

$$\overline{U}_0^s \simeq \overline{U}_0 + \left(\overline{H}_0 \overline{c}\right)^{1-\sigma} \left[\widehat{c}_0 + \frac{1}{2}(1-\sigma)(\widehat{c}_0)^2\right] \tag{A7b}$$

6.2.4 Second-order approximation of expected lifetime utility

Expected lifetime utility, V_t is given by the discounted infinite sum of $(A7a)$ and $(A7b)$, e.g.

$$V_t \simeq \underbrace{\overline{U}_0 + \left(\overline{H}_0 \overline{c}\right)^{1-\sigma} \left[\widehat{c}_0 + \frac{1}{2}(1-\sigma)(\widehat{c}_0)^2\right]}_{\overline{U}_0^s} + \qquad (A8)$$

$$E_0 \sum_{t=1}^{\infty} \beta^t \underbrace{\left\{ \overline{U}_t + \left(\overline{H}_0 \overline{c} \overline{\gamma}^t\right)^{1-\sigma} \left[(\widehat{c}_t + \sum_{s=0}^{t-1} \widehat{\gamma}_s) + \frac{1}{2}(1-\sigma)(\widehat{c}_t + \sum_{s=0}^{t-1} \widehat{\gamma}_s)^2 \right] \right\}}_{\overline{U}_t^s}$$

In the simulations $T = 300$ and the sample average for V is calculated using 1000 simulations.[25]

6.2.5 Welfare comparisons

Let $\overline{C}_t^A$ denote the contingent plan for per capita consumption associated with regime A and $\overline{C}_t^B$ the contingent plan for per capita consumption associated with regime B. We can then, following e.g. Lucas (1990), define ξ as the fraction of regime B's consumption process that a household would be willing to give up to be as well off under regime A as under B. Hence, we can write:

$$\begin{aligned} V_{(A)} &= E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \overline{U}_t \left[\overline{C}_t^B (1-\xi) \right] \right\} \\ &= E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(\overline{C}_t^B\right)^{1-\sigma}}{1-\sigma} (1-\xi)^{1-\sigma} \right] \right\} \\ &= (1-\xi)^{1-\sigma} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(\overline{C}_t^B\right)^{1-\sigma}}{1-\sigma} \right] \right\} \\ &= (1-\xi)^{1-\sigma} V_{(B)}. \qquad (A9) \end{aligned}$$

[25] Note that for our annual rate of time preference, $\beta = 0.964$, $V_T \simeq 0$ for $T = 300$. In contrast, studies using quarterly data with $\beta = 0.99$, $T = 1000$ is required.

Solving for ξ we then obtain:

$$\begin{aligned} V_{(A)} &= (1-\xi)^{1-\sigma} V_{(B)} \\ &\Rightarrow \ln(1-\xi) = \frac{1}{1-\sigma} \times \ln\left(\frac{V_{(A)}}{V_{(B)}}\right) \\ &\Rightarrow \xi \simeq \frac{1}{1-\sigma} \times \ln\left(\frac{V_{(B)}}{V_{(A)}}\right) \end{aligned} \quad (A.10)$$

where, $V_{(B)}$ and $V_{(A)}$ are calculated using the second-order approximation of welfare defined in $(A8)$ averaged over 1000 simulations.

Appendix Figure 1: Partial effects of changes in the parmeters on γ

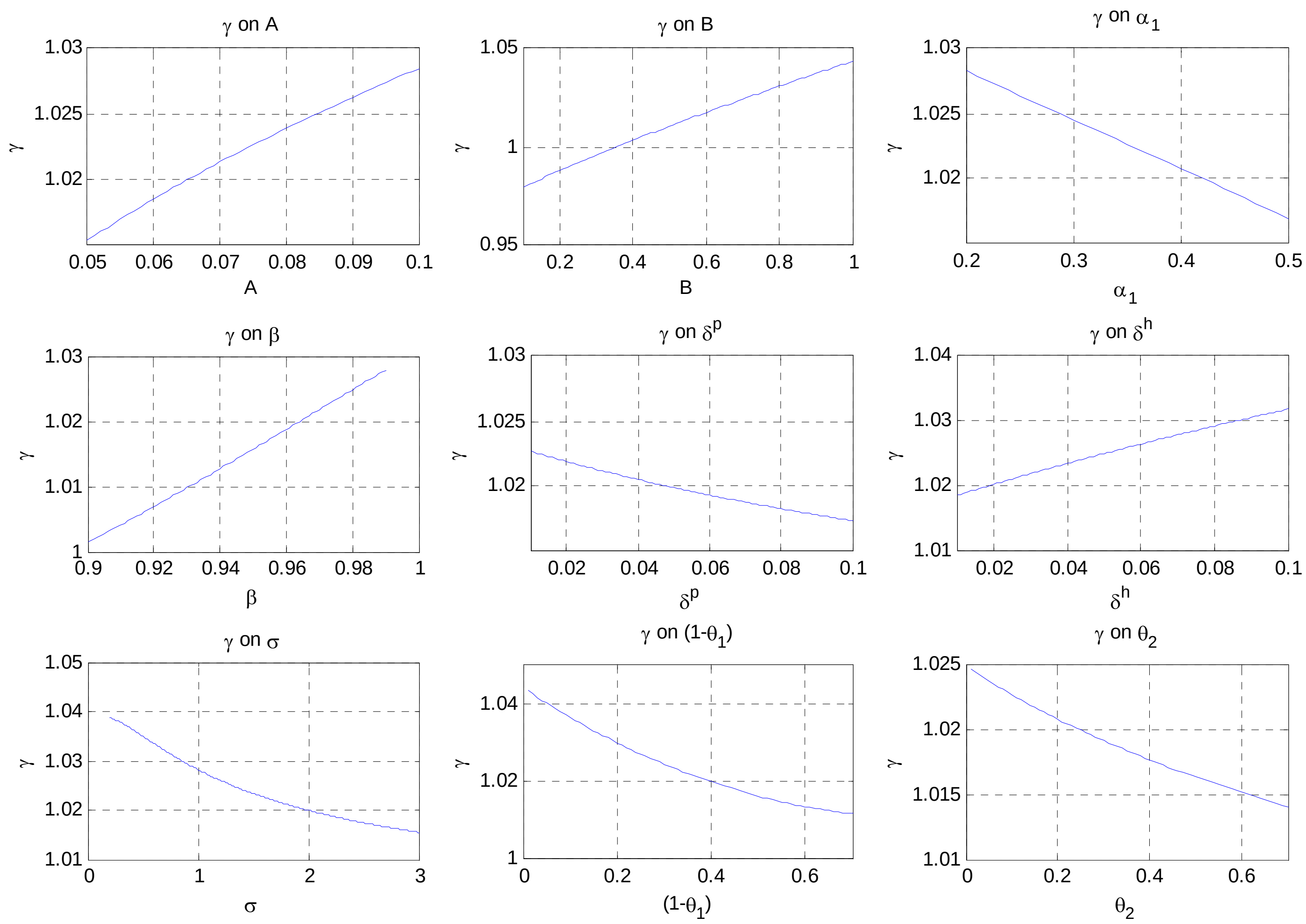

Figure 1: Data and model auto- and cross-correlation functions with 95% confidence bands

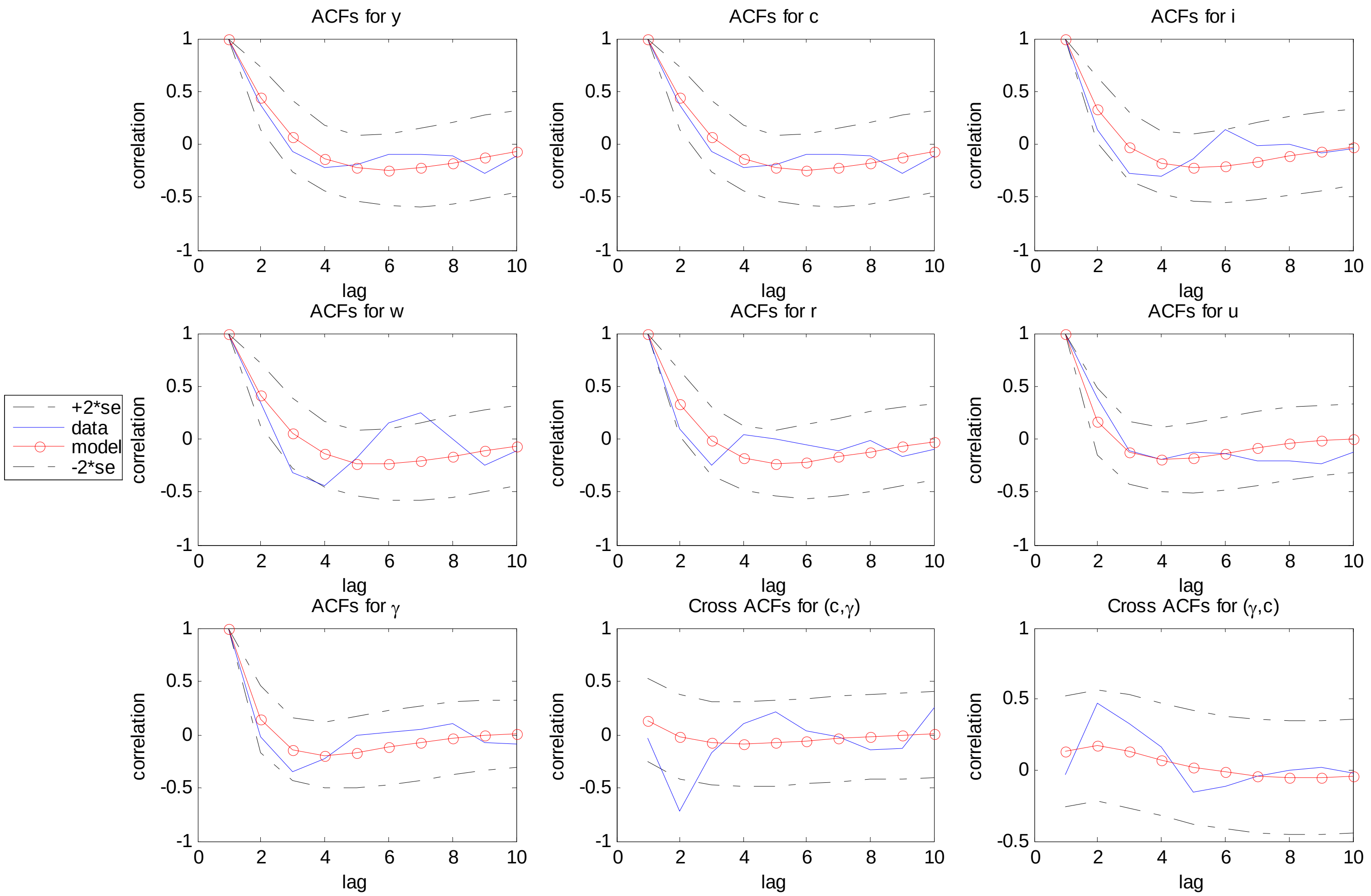

Figure 2: Effects of public education spending on steady-state growth and lifetime utility

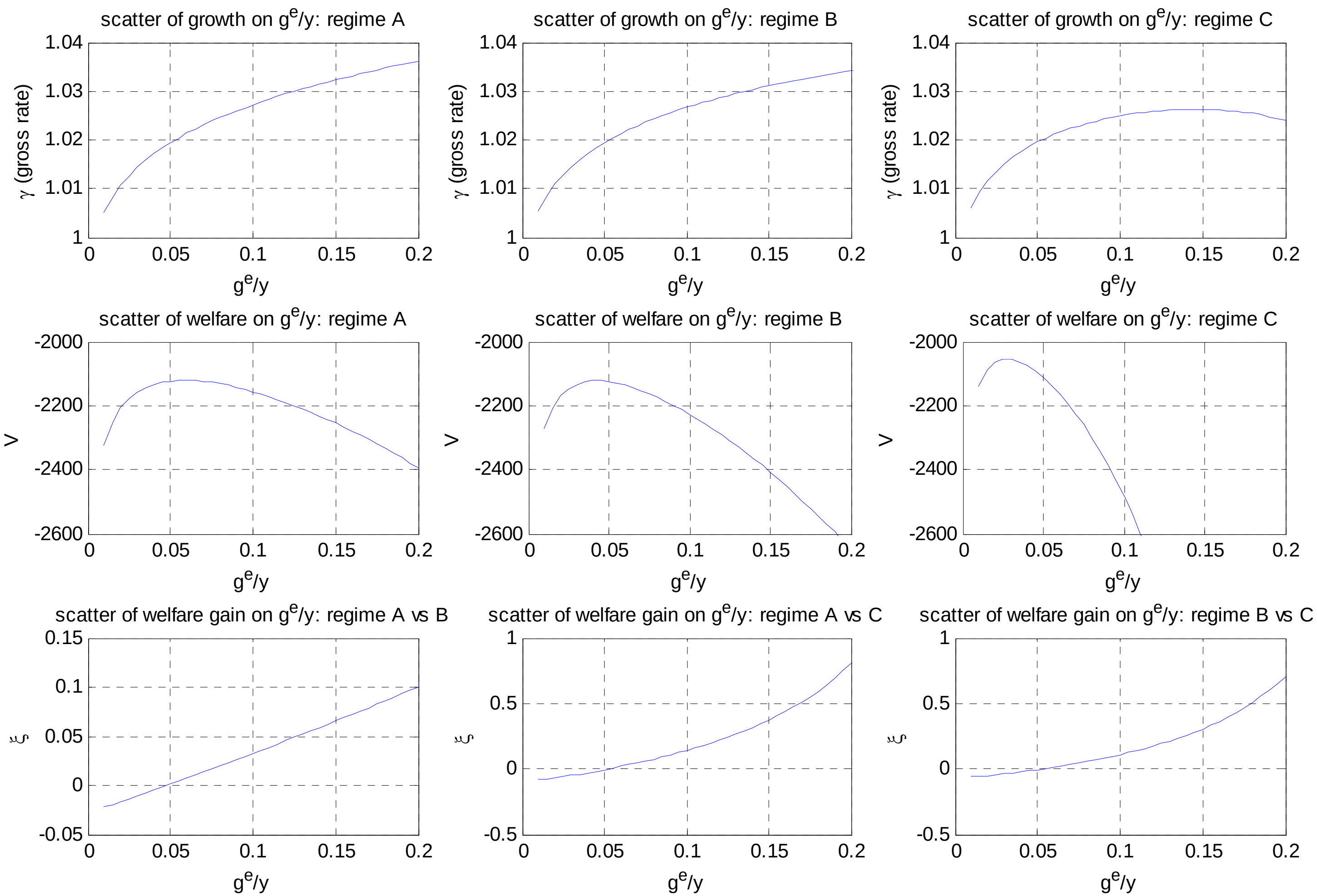

Figure 3: Steady-state effects of public education spending

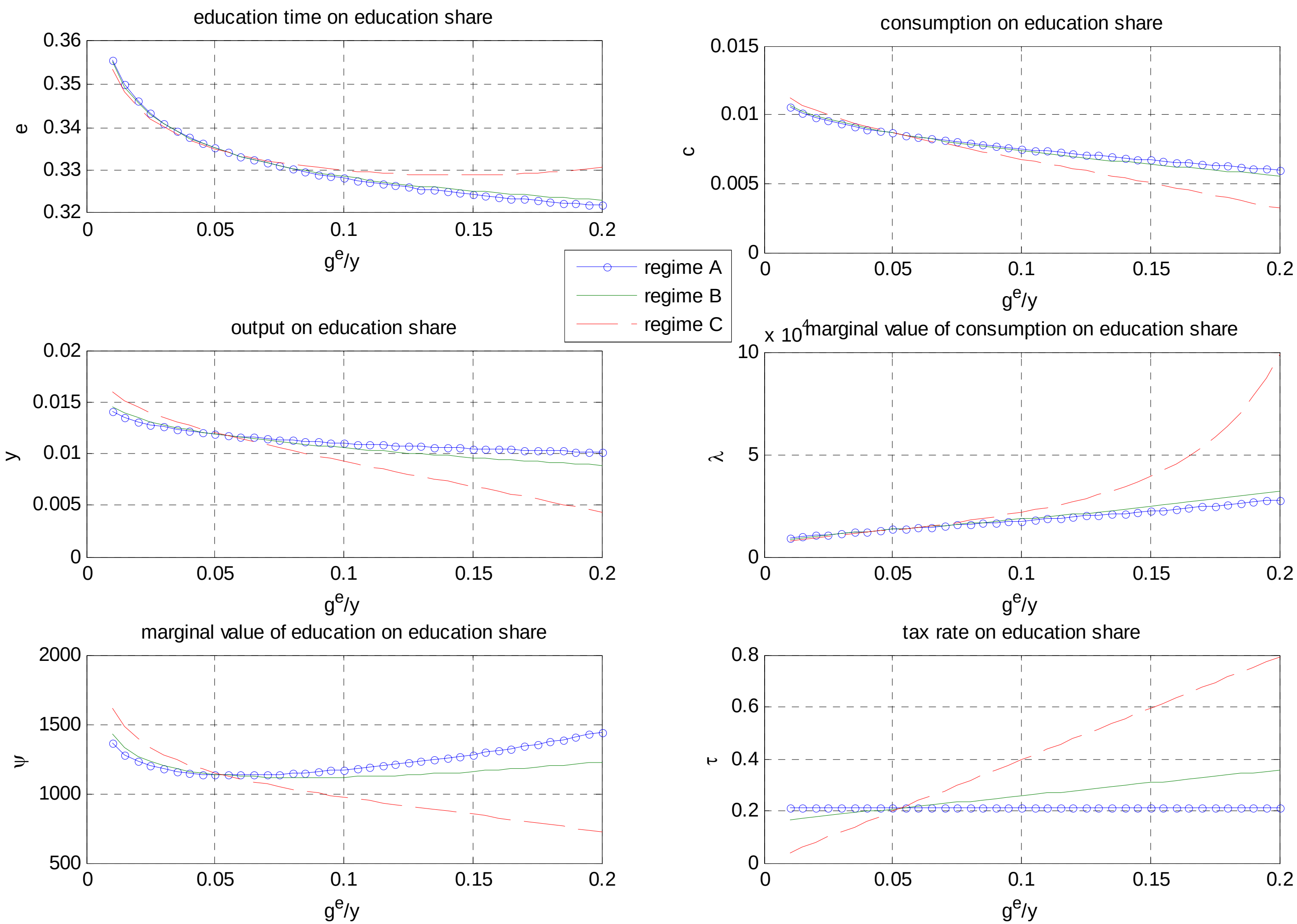

Figure 4a: Uncertainy and welfare across regimes

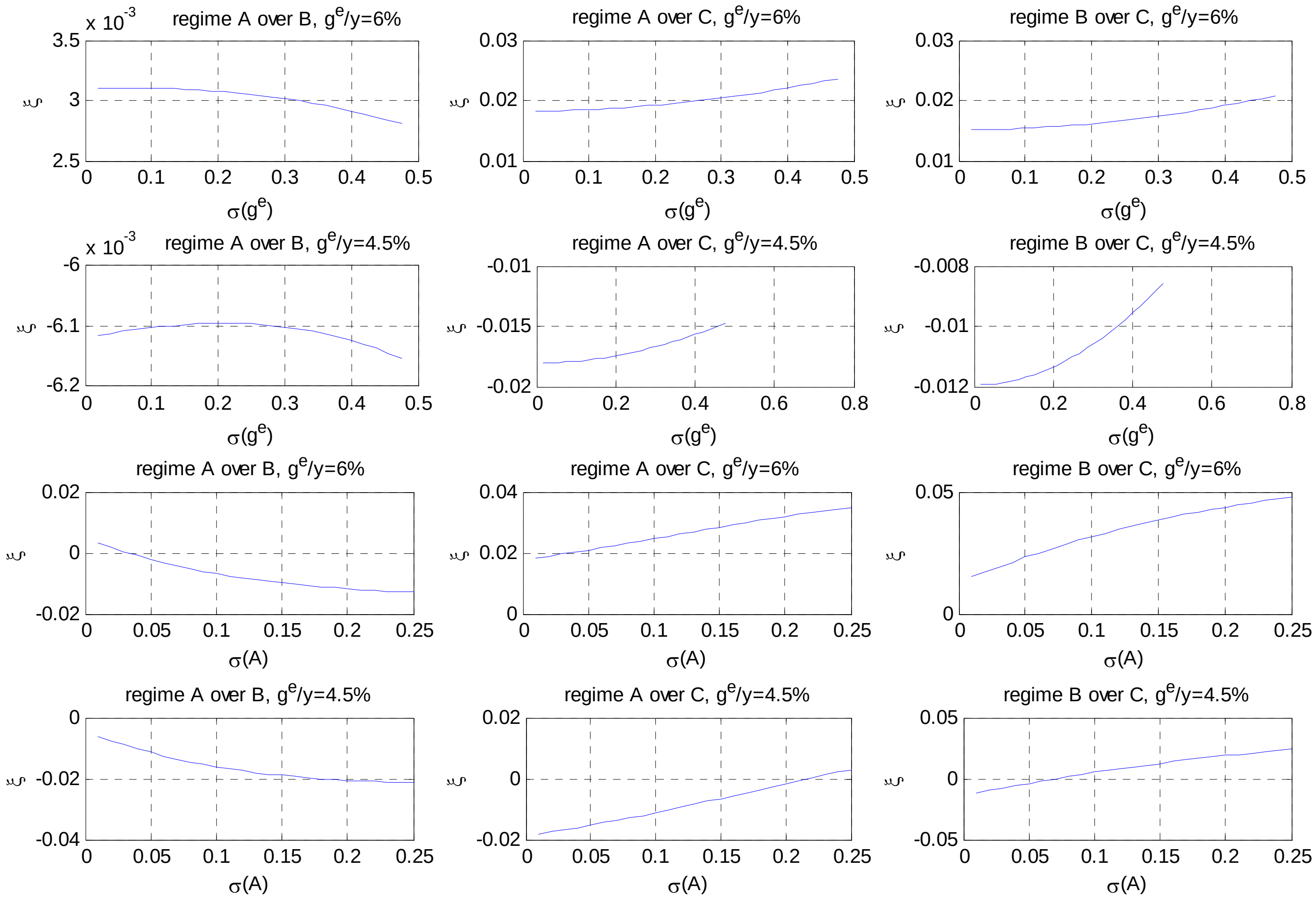

Figure 4b: Composition of welfare and uncertainty

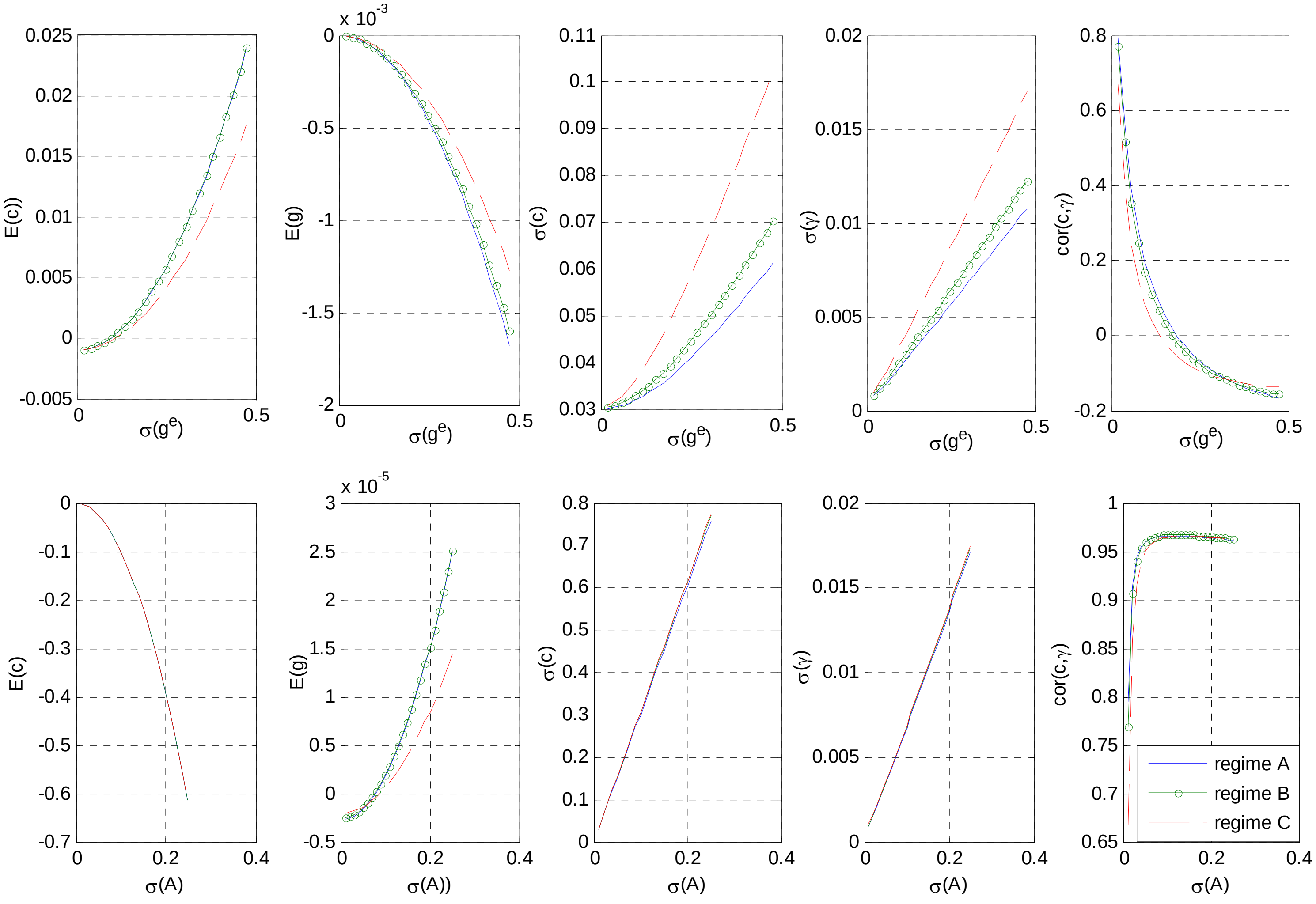

Figure 5a: Impulse responses to public education innovations

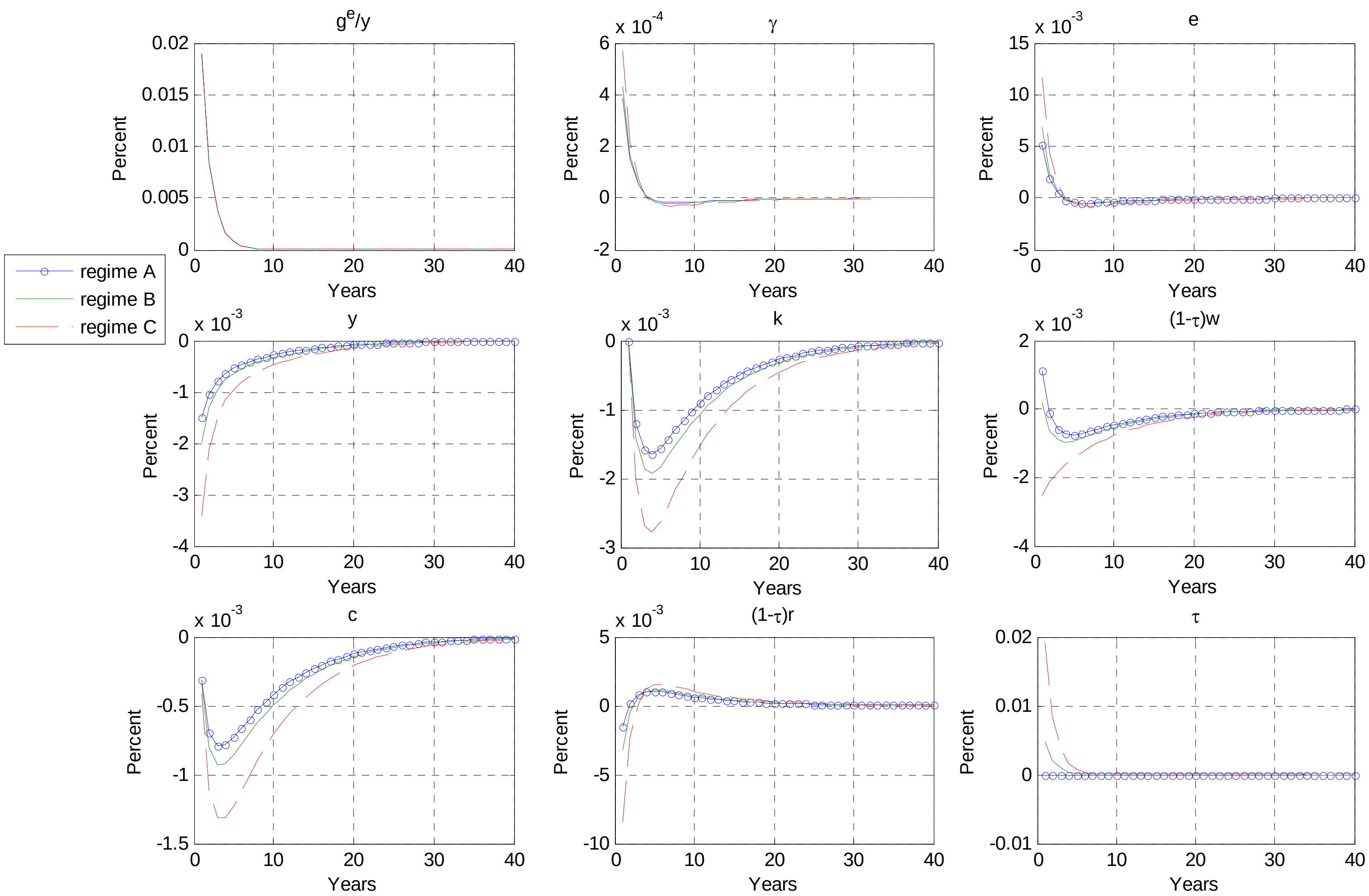

Figure 5b: Impulse responses to TFP innovations