# LÉVY PROCESSES, CHANGE OF MEASURE AND APPLICATIONS IN FINANCE

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## 1. Introduction

Definition of Lévy processes; basic examples: Brownian motion, Poisson process, Merton and Kou models, NIG and VG processes. The Lévy–Khintchine formula and the Lévy–Itô decomposition. Lévy martingales.

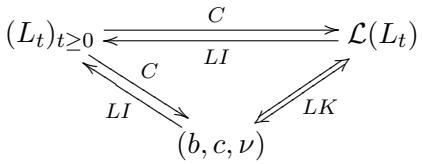


FIGURE 1. Commutative diagram of the relationship between a Lévy process $(L_t)_{t\geq 0}$, the law of the infinitely divisible random variable $\mathcal{L}(L_t)$ and the Lévy triplet $(b, c, \nu)$, demonstrating the role of the Lévy–Khintchine formula and the Lévy–Itô decomposition.

## 2. Modeling asset prices

Stochastic exponential (i.e. SDE) vs. natural exponential for modeling asset price processes; advantages and disadvantages. Examples: Brownian motion, Merton model.

## 3. Change of measure

The Radon–Nikodym theorem for functions. Girsanov's theorem for Lévy processes. Esscher transform (proof in this case). Examples: change of measure for Brownian motion and Lévy processes.

## 4. Incomplete markets

Description of the asset price process under the 'real-world' and the 'risk-neutral' measure. A simple equation for the 'market price of risk' tuple. Complete markets: Brownian motion, Poisson process. A simple incomplete model.

## 5. On the choice of numeraire

Examples from equity and interest rate markets where the choice of a suitable numeraire simplifies the valuation problem.

## 6. Option pricing by transform methods

Presentation of the basic ideas of Carr/Madan and Raible. Examples of option payoffs.

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