Asset Prices and Risk Sharing in Open Economies

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Introduction

- Asset pricing theory has mainly focused on closed economies.
- Consumption $=$ Endowment. Add preferences, and get asset prices.
- Standard model: Lucas (1978).

Introduction

- This paper focuses on open economies; economies that trade goods and assets internationally.
- There are **asset pricing** implications of the ability to trade internationally. We will see that they are important and depend on the size of the economy and its degree of home bias.
- Increasing globalization will make the focus on open economies more and more relevant.

Introduction

Any open-economy asset pricing model should:

1. generate realistic magnitudes for asset prices and returns,
2. generate reasonable exchange rate dynamics and international return correlations,
3. address the major international finance puzzles,
4. generate plausible international trade dynamics.

- This paper proposes a model that addresses all those issues.
- The puzzles to be addressed are the international risk sharing puzzle and the exchange rate disconnect puzzle.

The international risk sharing puzzle

In the absence of frictions in international asset markets, the following **no arbitrage** condition must hold:

$$\frac{M^*_{t+1}}{M_{t+1}} = \frac{E_{t+1}}{E_t}$$

where
M_{t+1} is the home country SDF,
M^*_{t+1} is the foreign country SDF, and
E_t is the real exchange rate between the two countries.

Real exchange rate changes are a wedge against perfect international risk sharing. Hence:

- High volatility of the real exchange rate = Low international risk sharing.

The international risk sharing puzzle

Beginning with the **no arbitrage** condition

$$\frac{M^*_{t+1}}{M_{t+1}} = \frac{E_{t+1}}{E_t}$$

and taking logs, and then unconditional variances of both sides, we get:

$$var(m^*_{t+1}) + var(m_{t+1}) - 2cov(m_{t+1}, m^*_{t+1}) = var(\Delta e_{t+1})$$

The international risk sharing puzzle

Consider magnitudes in

$$var(m^*_{t+1}) + var(m_{t+1}) - 2cov(m_{t+1}, m^*_{t+1}) = var(\Delta e_{t+1})$$

- High Sharpe ratios in asset markets tell us that SDFs are very volatile, by Hansen-Jagannathan bounds logic.
- Real exchange rate volatility is much lower: $\sigma(\Delta e_{t+1})$ is **around 10% - 15%** for major currency pairs.
- Therefore, *financial prices* tell us that $\rho(m_{t+1}, m^*_{t+1})$ must be **high**; **much risk sharing**.
- For $\sigma(m_{t+1}) = \sigma(m^*_{t+1}) = 50\%$ and $\sigma(\Delta e_{t+1}) = 10\%$, $\rho(m_{t+1}, m^*_{t+1}) = 0.98$.

The international risk sharing puzzle

Quantities tell a different story.

$$var(m^*_{t+1}) + var(m_{t+1}) - 2cov(m_{t+1}, m^*_{t+1}) = var(\Delta e_{t+1})$$

Consider standard power preferences: $m_{t+1} = \log\beta - \gamma\Delta c_{t+1}$.

- Then, $\sigma(m_{t+1}) = \gamma\sigma(\Delta c_{t+1})$. To get the equity premium, need high γ.
- Further, $\rho(m_{t+1}, m^*_{t+1}) = \rho(\Delta c_{t+1}, \Delta c^*_{t+1})$. Consumption growth is not very correlated across countries: $\rho(\Delta c_{t+1}, \Delta c^*_{t+1}) = \mathbf{0.2 - 0.6}$ for major country pairs.
- Quantities tell us that $\rho(m_{t+1}, m^*_{t+1})$ must be **low**; **poor risk sharing**.
- The implication is that the real exchange rate has to be **very volatile**.
- For $\gamma = 33$, $\sigma(m_{t+1}) = \sigma(m^*_{t+1}) = 1.5\%$ and $\rho(\Delta c_{t+1}, \Delta c^*_{t+1}) = 0.35$, $var(\Delta e_{t+1}) = 56\%$.

The international risk sharing puzzle

Hence...

- Brandt, Cochrane, Santa-Clara (2006), *"International risk sharing is better than you think, or exchange rates are too smooth"*.
- Two choices:
  - Accept the quantity story that $\rho(\Delta m_{t+1}, \Delta m^*_{t+1})$ is low and somehow explain why the extremely high theoretical real exchange volatility is not observed in the data, or
  - Accept that $\sigma(\Delta e_{t+1})$ is low and explain why $\rho(m_{t+1}, m^*_{t+1})$ is high **along with** low $\rho(\Delta c_{t+1}, \Delta c^*_{t+1})$, reconciling prices and quantities.
- This paper chooses the second explanation.

The exchange rate disconnect puzzle

- Recall that

$$\frac{M^*_{t+1}}{M_{t+1}} = \frac{E_{t+1}}{E_t}$$

- For power preferences, $\rho(\Delta e_{t+1}, \Delta c_{t+1} - \Delta c^*_{t+1}) = 1$, no matter what γ is.
- In the data, $\rho(\Delta e_{t+1}, \Delta c_{t+1} - \Delta c^*_{t+1}) \simeq 0$.
- Exchange rates appear to be disconnected from consumption growth!

Model overview

The model contains three main ingredients:

1. home bias in the consumption preferences,
2. external habits, i.e. time-varying risk aversion,
3. the ability to trade internationally in goods and assets.

All three are crucial.

Previous literature

- Habits: Sundaresan (1989), Abel (1990), Constantinides (1990), Ferson and Constantinides (1991), Campbell and Cochrane (1999), Menzly, Santos and Veronesi (2004), Santos and Veronesi (2006).
- Two country models with habits: Boldrin, Christiano and Fisher (2001), Moore and Roche (2006), Verdelhan (2007).
- Two country models with Epstein-Zin preferences: Colacito and Croce (2007, 2008), Bansal and Shaliastovich (2007).
- Terms of trade effects: Lucas (1982), Cole and Obstfeld (1991), Zapatero (1995), Serrat (2001), Pavlova and Rigobon (2007, 2008).

- Two countries (Home and Foreign), each populated by a single risk-averse representaive agent who maximizes $t = 0$ discounted expected utility and has an infinite horizon.
- There two goods in the global economy, the home good and the foreign good. The global endowment of the home and the foreign good is the Itô process $\{\widetilde{X}_t\}$ and $\{\widetilde{Y}_t\}$, respectively.
- The home country (agent) is endowed with the global endowment of the home good and receives no foreign good. Conversely, the foreign country receives the global endowment of the foreign good.

- Trade in goods is frictionless; the law of one price holds for each individual good.
- Denote by Q_t the price of the home good and by Q_t^* the price of the foreign good. WLOG, set the home good as numeraire, so $Q_t \equiv 1, \forall t \in [0, \infty)$.
- Financial markets are complete and there are no frictions in the international trade of financial assets, so $\frac{M_{t+1}^*}{M_{t+1}} = \frac{E_{t+1}}{E_t}$ holds.
- Agents have rational expectations.

The model

Preferences

- The representative agent of the home country has log **external habit** preferences

$$u(C_t, H_t) = \log(C_t - H_t)$$

- The habit is on the *consumption basket* C_t defined as

$$C_t \equiv X_t^{\alpha} Y_t^{1-\alpha}$$

- The elasticity of substitution between the two goods is 1.
- For $\alpha > 0.5$ there is home bias in preferences; the home agent cares more about the local good that the foreign one. No bias when $\alpha = 0.5$, complete bias when $\alpha = 1$.

The model

Preferences

- Standard model for external habits: Campbell and Cochrane (1999).
- Define the surplus consumption ratio $S_t \equiv \frac{C_t - H_t}{C_t}$.
- The process for the log of S_t is:

$$\Delta s_{t+1} = (1-\phi)(\bar{s} - s_t) + \lambda(s_t)(\Delta c_{t+1} - E_t(\Delta c_{t+1}))$$

  where $\lambda(s_t)$ is the **sensitivity** of the habit to consumption growth shocks.

  The habit adjusts slowly to the history of consumption. In expansions, S_t is high; in recessions, S_t is low.
- For power utility, the time-varying risk aversion will be $\frac{\gamma}{S_t}$.
  Countercyclical risk aversion.

The model

Preferences

$$\Delta s_{t+1} = (1-\phi)(\bar{s} - s_t) + \lambda(s_t)(\Delta c_{t+1} - E_t(\Delta c_{t+1}))$$

- The sensitivity is decreasing in s_t, i.e. the sensitivity is high in recessions. **Consumption growth shocks matter more in recessions.**
- That buys us time variation in risk premia, since the market price of risk is

$$\sigma_t(m_{t+1}) = \gamma(1 + \lambda(s_t))\sigma_c$$

  and this is the Hansen-Jagannathan lower bound for conditional Sharpe ratios. **Countercyclical risk premia**.
- Two terms: one that accounts for **risk aversion**: $\gamma(1 + \lambda(s_t))$, one that accounts for **risk**: σ_c.

The model

Preferences

- Recall that time-varying risk aversion is $\frac{\gamma}{S_t}$. Here, log preferences, so $\gamma = 1$.
- Assume Menzly, Santos, Veronesi (2004) external habits. $G_t = \frac{1}{S_t}$ solves

$$\frac{dG_t}{G_t} = k\left(\frac{\bar{G} - G_t}{G_t}\right)dt - \delta\left(\frac{G_t - l}{G_t}\right)\left(\frac{dC_t}{C_t} - E_t\left(\frac{dC_t}{C_t}\right)\right)$$

- G_t is also the time-varying RRA coefficient.
- G_t is driven by consumption growth shocks and has a lower bound l.
- The sensitivity of the growth rate of G_t is $\delta\left(\frac{G_t - l}{G_t}\right) \geq 0$. It is decreasing in G_t; **consumption growth shocks matter more in recessions.**

- The foreign representative agent will also have MSV external habit preferences.
- The only difference is the consumption basket:

$$C_t^* \equiv \left(X_t^*\right)^{\alpha^*} \left(Y_t^*\right)^{1-\alpha^*}$$

- Home bias for the foreign agent means $\alpha^* < 0.5$.

The model

Endowments

- The two endowment processes $\widetilde{X}$ and $\widetilde{Y}$ are driven by the Brownian shocks dB_t^X and dB_t^Y, respectively, where $corr(dB_t^X, dB_t^Y) = \rho^{XY}$.
- For the real exchange rate to be stationary, the ratio $z_t = \frac{\widetilde{Y}_t}{\widetilde{X}_t}$ has to be stationary. If not, one country will dominate the other as $T \longrightarrow \infty$.
- Therefore, consider the log of endowment ratio $z_t = \frac{\widetilde{Y}_t}{\widetilde{X}_t}$. It follows a continuous-time AR(1) process:

$$d \log z_t = \theta(\log \bar{z} - \log z_t)dt + \sigma^z dB_t^z$$

  where dB_t^z is a linear function of dB_t^X and dB_t^Y.
- For $\theta = 0$, no cointegration, so the real exchange rate is non-stationary.

The model
Endowments

- The endowment processes $\widetilde{X}$ and $\widetilde{Y}$ solve, respectively

$$d \log \widetilde{X}_t = \left[\mu - \psi\theta(\log \bar{z} - \log z_t)\right] dt + \sigma^X dB_t^X \quad \text{and}$$
$$d \log \widetilde{Y}_t = \left[\mu + (1-\psi)\theta(\log \bar{z} - \log z_t)\right] dt + \sigma^Y dB_t^Y$$

- The unconditional mean of both processes is μ.
- Parameter ψ controls error-correction:
  - for $\psi = 0$, $\widetilde{Y}$ error-corrects,
  - for $\psi = 1$, $\widetilde{X}$ error-corrects,
  - for $\psi \in (0, 1)$, both processes error-correct.
- Note that both endowment processes are **homoskedastic**.

The model

Prices and exchange rates

- Given that the home consumption basket is $C = X^{\alpha} Y^{1-\alpha}$, the home price index is:

$$P_t = \left(\frac{Q_t}{\alpha}\right)^{\alpha} \left(\frac{Q_t^*}{1-\alpha}\right)^{1-\alpha}$$

  and similarly for the foreign price index.

- The real exchange rate is:

$$E_t = \frac{P_t^*}{P_t} = \frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(\alpha^*)^{\alpha^*}(1-\alpha^*)^{1-\alpha^*}} (Q_t^*)^{\alpha-\alpha^*}$$

  using the fact that $Q_t = 1, \forall t \in [0, \infty)$.

- When preferences are identical ($\alpha = \alpha^*$), consumption baskets are identical ($C = C^*$), so $E = 1$ (PPP): perfect risk sharing.
- When $\alpha \neq \alpha^*$, then $C \neq C^*$, so E varies with changes in the relative price Q^*.

The planner's problem

- Since markets are complete, the competitive equilibrium solution is PO, so we can solve the planner's problem.

$$\max_{\{X_t, Y_t, X_t^*, Y_t^*\}} E_0 \left[\int_0^\infty e^{-\rho t} \left(\lambda \log(C_t - H_t) + \lambda^* \log(C_t^* - H_t^*) \right) dt \right]$$

s.t. the market clearing constraints $X_t + X_t^* = \widetilde{X}_t$ and $Y_t + Y_t^* = \widetilde{Y}_t$.

- For the planner's problem solution to be equivalent to the CE solution, the welfare weights λ and $\lambda^* = 1 - \lambda$ implicitly depend on the budget constraint of each country. **The welfare weights are endogenous**.
- We will see that, in this model, the welfare weights are connected to the initial wealth of each country.

Consumption

For the home agent:

$$X_t = \frac{\alpha\lambda}{\alpha\lambda + \alpha^*\lambda^*\left(\frac{G_t^*}{G_t}\right)}\widetilde{X}_t\ , \qquad Y_t = \frac{(1-\alpha)\lambda}{(1-\alpha)\lambda + (1-\alpha^*)\lambda^*\left(\frac{G_t^*}{G_t}\right)}\widetilde{Y}_t$$

and for the foreign agent:

$$X_t^* = \frac{\alpha^*\lambda^*\left(\frac{G_t^*}{G_t}\right)}{\alpha\lambda + \alpha^*\lambda^*\left(\frac{G_t^*}{G_t}\right)}\widetilde{X}_t\ , \qquad Y_t^* = \frac{(1-\alpha^*)\lambda^*\left(\frac{G_t^*}{G_t}\right)}{(1-\alpha)\lambda + (1-\alpha^*)\lambda^*\left(\frac{G_t^*}{G_t}\right)}\widetilde{Y}_t$$

Note that both X_t and Y_t are decreasing in $\frac{G_t^*}{G_t}$ and both X_t^* and Y_t^* are incrasing in $\frac{G_t^*}{G_t}$. Ceteris paribus, **the more conditionally risk averse country consumes more**.

Consumption

Home and foreign consumption is a function of the two endowments $\widetilde{X}_t$ and $\widetilde{Y}_t$ and of the relative conditional risk aversion of the two countries $\frac{G_t^*}{G_t}$.

$$
\begin{aligned}
C_t &= \frac{\lambda \alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\left(\alpha\lambda + \alpha^*\lambda^*\left(\frac{G_t^*}{G_t}\right)\right)^{\alpha}\left((1-\alpha)\lambda + (1-\alpha^*)\lambda^*\left(\frac{G_t^*}{G_t}\right)\right)^{1-\alpha}} \widetilde{X}_t^{\alpha}\widetilde{Y}_t^{1-\alpha} \\
C_t^* &= \frac{\lambda^*\left(\alpha^*\right)^{\alpha^*}\left(1-\alpha^*\right)^{1-\alpha^*}\left(\frac{G_t^*}{G_t}\right)}{\left(\alpha\lambda + \alpha^*\lambda^*\left(\frac{G_t^*}{G_t}\right)\right)^{\alpha^*}\left((1-\alpha)\lambda + (1-\alpha^*)\lambda^*\left(\frac{G_t^*}{G_t}\right)\right)^{1-\alpha^*}} \widetilde{X}_t^{\alpha^*}\widetilde{Y}_t^{1-\alpha^*}
\end{aligned}
$$

Consumption is increasing both endowments $\widetilde{X}_t$ and $\widetilde{Y}_t$, with C_t more sensitive to $\widetilde{X}_t$ and C_t^* more sensitive to $\widetilde{Y}_t$ when $\alpha > \alpha^*$.

Consumption

Since consumption and habit are jointly determined, we need to solve for the home and foreign consumption processes as functions of the two exogenous shocks dB_t^X and dB_t^Y.
Let

$$\begin{aligned} \frac{dC_t}{C_t} - E_t\left(\frac{dC_t}{C_t}\right) &= \sigma_t^{CX} dB_t^X + \sigma_t^{CY} dB_t^Y \quad \text{and} \\ \frac{dC_t^*}{C_t^*} - E_t\left(\frac{dC_t^*}{C_t^*}\right) &= \sigma_t^{C^*X} dB_t^X + \sigma_t^{C^*Y} dB_t^Y \end{aligned}$$

Consumption

For the home consumption process:

$$\sigma_t^{CX} = \frac{1}{D_t}\left(\alpha + (\alpha k_t^* + \alpha^* k_t)\,\delta\left(\frac{G_t^* - I}{G_t^*}\right)\right)\sigma^X$$

$$\sigma_t^{CY} = \frac{1}{D_t}\left((1-\alpha) + ((1-\alpha)k_t^* + (1-\alpha^*)k_t)\,\delta\left(\frac{G_t^* - I}{G_t^*}\right)\right)\sigma^Y$$

and for the foreign consumption process:

$$\sigma_t^{C^*X} = \frac{1}{D_t}\left(\alpha^* + (\alpha k_t^* + \alpha^* k_t)\,\delta\left(\frac{G_t - I}{G_t}\right)\right)\sigma^X$$

$$\sigma_t^{C^*Y} = \frac{1}{D_t}\left((1-\alpha^*) + ((1-\alpha)k_t^* + (1-\alpha^*)k_t)\,\delta\left(\frac{G_t - I}{G_t}\right)\right)\sigma^Y$$

where k_t, k_t^* and $D_t > 0$ are functions of of G_t and G_t^*. For $0 < \alpha^* < \alpha < 1$, it holds that $0 < k_t < 1$ and $0 < k_t^* < 1, \forall t \in [0, \infty)$.

Consumption

- We see that each country's consumption growth volatility is increasing in the *other country's* conditional risk aversion.

- The *relatively* more risk averse country has *relatively* lower conditional consumption growth volatility. Why?
- **Countries share risk**: the more risk averse country wants safe consumption, the less risk averse country can handle risky consumption.
- The global endowment risk is taken by the country that is less averse to it.

- **Risk sharing by international trade in goods and assets.**

Solving the risk sharing puzzle

- Recall that the market price of risk is the product of
  - sensitivity to consumption risk (conditional risk aversion)
  - consumption risk (conditional volatility of consumption growth)
- In this model, high conditional risk aversion is multiplied by low conditional consumption growth volatility, and vice versa.
- Then, the market prices of risk for the two countries are very close to each other.
- The two SDFs are very highly correlated!

Solving the risk sharing puzzle

- The marginal utility of home consumption is $\Lambda_t = e^{-\rho t} \frac{G_t}{C_t}$, so the home log pricing kernel is

$$\begin{aligned} d \log \Lambda_t &= -\rho dt + d \log G_t - d \log C_t \\ &= drift - \left(1 + \delta \frac{G_t - l}{G_t}\right) \left(\sigma_t^{C\prime} dB_t\right) \end{aligned}$$

with $\sigma_t^{C\prime} dB_t$ being (roughly) proportional to $\delta \left(\frac{G_t^* - l}{G_t^*}\right)$.

- So, the first order term is a term $\delta^2 \left(\frac{G_t - l}{G_t}\right) \left(\frac{G_t^* - l}{G_t^*}\right) f(k_t, k_t^*)$
- The foreign SDF has **exactly** the same first order term.
- EXTREMELY correlated SDFs: $corr(d \log \Lambda_t, d \log \Lambda_t^*)$ is very close to 1.
- Consumption growths can diverge: there is no sensitivity term to balance out differences in conditional consumption growth volatility, so $corr(d \log C_t, d \log C_t^*) < 1$.

The terms of trade and the real exchange rate

The terms of trade are

$$Q_t^* = \frac{(1-\alpha)\lambda + (1-\alpha^*)\lambda^* \left(\frac{G_t^*}{G_t}\right)}{\alpha\lambda + \alpha^*\lambda^* \left(\frac{G_t^*}{G_t}\right)} \frac{\widetilde{X}_t}{\widetilde{Y}_t}$$

so the real exchange rate is

$$E_t = \frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{(\alpha^*)^{\alpha^*} (1-\alpha^*)^{1-\alpha^*}} \left(\frac{(1-\alpha)\lambda + (1-\alpha^*)\lambda^* \left(\frac{G_t^*}{G_t}\right)}{\alpha\lambda + \alpha^*\lambda^* \left(\frac{G_t^*}{G_t}\right)} \right)^{\alpha-\alpha^*} \left(\frac{\widetilde{X}_t}{\widetilde{Y}_t}\right)^{\alpha-\alpha^*}$$

If $\alpha > \alpha^*$, Q_t^* (and so E_t) is increasing in $\frac{G_t^*}{G_t}$: when $\frac{G_t^*}{G_t}$ is high, the foreign consumption demand is high and, in the presence of home bias, it is mostly expressed as demand for the foreign good, increasing its relative price.

- Note that changes $\frac{G_t^*}{G_t}$ tend to reinforce the effect of changes in $\frac{\widetilde{X}_t}{\widetilde{Y}_t}$.

International trade

The home net exports ratio (value of net exports over the value of the endowment) is

$$NX_t = \frac{\widetilde{X}_t - C_t P_t}{\widetilde{X}_t} = 1 - \frac{\lambda}{\alpha\lambda + \alpha^*\lambda^* \left(\frac{G_t^*}{G_t}\right)}$$

Home net exports are increasing in $\frac{G_t^*}{G_t}$: ceteris paribus, the less conditionally risk averse country consumes less and exports more.

Wealth and welfare weights

Home and foreign country wealth, W_t and W_t^*, expressed in units of the numeraire good, are connected by

$$\frac{W_t}{W_t^*} = \frac{\lambda}{\lambda^*} \frac{k\bar{G} + \rho G_t}{k\bar{G} + \rho G_t^*}$$

where $\lambda = \frac{\alpha^* \left(k\bar{G} + \rho G_0^*\right)}{(1-\alpha)\left(k\bar{G} + \rho G_0\right) + \alpha^* \left(k\bar{G} + \rho G_0^*\right)}$ and $\lambda^* = 1 - \lambda$.
Initial home wealth, as a proportion to global wealth, is

$$\frac{W_0}{W_0 + W_0^*} = \frac{\alpha^*}{1 - \alpha + \alpha^*}$$

Note that for $G_0 = G_0^*$

$$\lambda = \frac{\alpha^*}{1 + \alpha^* - \alpha} = \frac{W_0}{W_0 + W_0^*}$$

Wealth and welfare weights

$$\frac{W_0}{W_0 + W_0^*} = \frac{\alpha^*}{1 - \alpha + \alpha^*}$$

Note that initial home wealth is increasing in both α and α^* and that:

- if $\alpha = 1$ and $\alpha^* > 0$, i.e. complete home bias in the home country, then the home country has all the initial wealth,
- if $\alpha < 1$ and $\alpha^* = 0$, i.e. complete home bias in the foreign country, then the foreign country has all the initial wealth,
- if $a = 1$ and $a^* = 0$, i.e. both countries are completely home biased, then the initial wealth ratio is indeterminate.

Asset prices and returns

- So far, we have considered complete markets without explicitly specifying the assets in which the agents can invest.
- Consider 4 assets: the **home TWP**, the **foreign TWP**, the **home bond** and the **foreign bond**, with prices V_t, V_t^*, B_t^* and B_t^*, respectively. The price of each asset is expressed in units of the local good.
- The home TWP portfolio is the claim to the future home endowment flows $\{\widetilde{X}\}$, while the home bond is a locally riskless asset in terms of the home good and has price process $dB_t = r_t^f B_t dt$. Similarly for the foreign assets.

Asset prices and returns

The price-dividend ratio of the home TWP is

$$\frac{V_t}{\widetilde{X}_t} = \frac{1}{\rho}\left(\frac{k}{\rho+k}\frac{(\alpha\lambda+\alpha^*\lambda^*)\,\bar{G}}{\alpha\lambda G_t+\alpha^*\lambda^* G_t^*}+\frac{\rho}{\rho+k}\right)$$

Compare with the Menzly, Santos, Veronesi (2004) closed economy solution

$$\frac{V_t}{\widetilde{X}_t} = \frac{1}{\rho}\left(\frac{k}{\rho+k}\frac{\bar{G}}{G_t}+\frac{\rho}{\rho+k}\right)$$

Foreign risk aversion matters for domestic asset prices!

The price-dividend ratio of the foreign TWP is

$$\frac{V_t^*}{\widetilde{Y}_t} = \frac{1}{\rho}\left(\frac{k}{\rho+k}\frac{\left((1-\alpha)\lambda+(1-\alpha^*)\lambda^*\right)\bar{G}}{(1-\alpha)\lambda G_t+(1-\alpha^*)\lambda^* G_t^*}+\frac{\rho}{\rho+k}\right)$$

Asset prices and returns

$$\frac{V_t}{\widetilde{X}_t} = \frac{1}{\rho}\left(\frac{k}{\rho+k}\frac{(\alpha\lambda+\alpha^*\lambda^*)\bar{G}}{\alpha\lambda G_t+\alpha^*\lambda^* G_t^*}+\frac{\rho}{\rho+k}\right)$$

Domestic asset prices are more affected by G_t^* when

1. the foreign country is relatively wealthy (high λ^*)
2. the foreign country has a strong preference for home goods - and, thus, assets (high α^*).

- Makes sense: the home TW portfolio is a claim on home good flows, which are getting consumed by both the home and the foreign agent, so the preferences and the wealth of both should matter for its price.

Asset prices and returns

Taking this further, which countries' **risk preferences** will matter for international asset prices?

The model says...

1. Big and wealthy countries.
2. Countries that import a lot.

Which countries' asset prices will be heavily affected by world **risk preferences**?

The model says...

1. Small and poor countries.
2. Countries with export-led economies.

Those are **multiplicative** effects!

Asset prices and returns

The excess return of the home TWP, in terms of the home good, is

$$dR_t = \left(\boldsymbol{\sigma}_t^{\Xi\prime} \mathbf{C} \boldsymbol{\sigma}_t^R \right) dt + \boldsymbol{\sigma}_t^{R\prime} dB_t$$

where $\boldsymbol{\sigma}_t^{\Xi}$ is the market price of home good risk

$$\boldsymbol{\sigma}_t^{\Xi} = \sigma^X \mathbf{e}_1 - \left[\frac{\alpha\lambda G_t}{\alpha\lambda G_t + \alpha^*\lambda^* G_t^*} \boldsymbol{\sigma}_t^G + \frac{\alpha^*\lambda^* G_t^*}{\alpha\lambda G_t + \alpha^*\lambda^* G_t^*} \boldsymbol{\sigma}_t^{G^*} \right]$$

and $\boldsymbol{\sigma}_t^R$ is the loading of home TWP returns on endowment risk

$$\begin{aligned} \boldsymbol{\sigma}_t^R &= \sigma^X \mathbf{e}_1 - \frac{(\alpha\lambda + \alpha^*\lambda^*) k \bar{G}}{(\alpha\lambda + \alpha^*\lambda^*) k \bar{G} + \rho \left(\alpha\lambda G_t + \alpha^*\lambda^* G_t^* \right)} \times \\ & \left[\frac{\alpha\lambda G_t}{\alpha\lambda G_t + \alpha^*\lambda^* G_t^*} \boldsymbol{\sigma}_t^G + \frac{\alpha^*\lambda^* G_t^*}{\alpha\lambda G_t + \alpha^*\lambda^* G_t^*} \boldsymbol{\sigma}_t^{G^*} \right] \end{aligned}$$

Asset prices and exchange rates

$$E_t = c \left(\frac{\frac{V_t}{\widetilde{X}_t} - \frac{1}{\rho+k}}{\frac{V_t^*}{\widetilde{Y}_t} - \frac{1}{\rho+k}} \right)^{\alpha-\alpha^*} \left(\frac{\widetilde{X}_t}{\widetilde{Y}_t} \right)^{\alpha-\alpha^*}$$

where c is a constant.

- Without habits, the real exchange rate only depends on relative endowments: $\frac{\widetilde{X}_t}{\widetilde{Y}_t}$
- Adding habits, the P/D ratios become time-varying, so changes in expected returns also matter.

Equilibrium portfolios

- Four assets, two shocks: dB^X and dB^Y.
- Expessing returns in terms of the numeraire, 3 risky assets (home TWP, foreign TWP and foreign bond) and 1 riskless asset (home bond).
- 1 of the risky assets is redundant: WLOG, set the foreign bond to be the redundant asset.

Equilibrium portfolios

For $\alpha \neq \alpha^*$, the home country equilibrium portfolio weights are

$$x_t^V = (1-\alpha^*) \frac{(\alpha\lambda + \alpha^*\lambda^*)k\bar{G} + \rho\left(\alpha\lambda G_t + \alpha^*\lambda^* G_t^*\right)}{(\alpha-\alpha^*)\,\lambda\left(k\bar{G} + \rho G_t\right)}\ ,\ x_t^{V^*} = 1 - x_t^V$$

while the foreign country equilibrium portfolio weights are

$$x_t^{*V} = -(1-\alpha) \frac{(\alpha\lambda + \alpha^*\lambda^*)k\bar{G} + \rho\left(\alpha\lambda G_t + \alpha^*\lambda^* G_t^*\right)}{(\alpha-\alpha^*)\,\lambda^*\left(k\bar{G} + \rho G_t^*\right)}\ ,\ x_t^{*V^*} = 1 - x_t^{*V}$$

1. If $0 < \alpha^* < \alpha < 1$, $x_t^V > 1$ and $x_t^{V^*} < 0$, i.e. the home agent holds a long-short portfolio. The same is true for the foreign agent. Extreme home bias.
2. Home leverage is decreasing in G_t and increasing in G_t^*: we know that when $\frac{G_t}{G_t^*}$ increases, the home agent wants low consumption growth volatility, so she deleverages.

Calibration

Parameter	Value
α	0.96
α^*	0.12
ρ	0.04
k	0.16
δ	65
l	18
$\bar{G}$	34
$\bar{z}$	1
θ	0.2
μ	0.02
ψ	0.5
σ^W	0.007
σ^z	0.020
ϕ	0.084

Calibration

Moment	**Model**	**Data**
US welfare weight (λ)	0.75	-
US endowment growth mean	*2.02%*	*1.48%*
US endowment growth std	*1.54%*	*1.51%*
UK endowment growth mean	*2.02%*	*2.53%*
UK endowment growth std	*3.97%*	*3.96%*
US/UK endowment growth corr	*0.134*	*0.155*
US consumption growth mean	2.02%	1.80%
US consumption growth std	1.52%	1.21%
UK consumption growth mean	2.02%	2.59%
UK consumption growth std	2.74%	2.01%
US/UK consumption growth corr	0.59	0.37
US/UK pricing kernel corr	0.97	-

Calibration

Moment	Model	Data
real exchange rate growth std	9.82%	10.18%
US terms of trade growth std	11.69%	3.07%
UK terms of trade growth std	15.34%	2.83%
US openness mean	0.08	0.38
UK openness mean	0.24	1.21
$corr(\Delta e_{t+1}, \Delta c^*_{t+1} - \Delta c_{t+1})$	0.10	0.01

Calibration

Moment	Model	Data
US excess return mean	7.42%	7.84%
US excess return std	23.97%	15.73%
UK excess return mean	9.79%	8.22%
UK excess return std	27.81%	16.52%
US/UK excess return corr	0.94	0.69
US Sharpe ratio	0.310	0.499
UK Sharpe ratio	0.352	0.497
US pricing kernel std	36.57%	-
UK pricing kernel std	40.03%	-
$corr(R^{e,US}_{t+1}, \Delta e_{t+1})$	-0.22	0.01
$corr(R^{e,UK}_{t+1}, \Delta e_{t+1})$	-0.50	-0.12

Conclusion

- The model generates economically intuitive closed-from results and convicingly solves two major international finance puzzles.
- The preliminary calibration results show that the model can capture many of the key moments in the data.
- Limitations and suggestions for future work: there is a role for non-tradable goods and incomplete markets.

Thank you for coming to the presentation!