Pricing Two Trees When Trees and Investors are Heterogeneous

Nicole Branger[1], Christian Schlag[2], Lue Wu[2]

[1]University of Münster

[2]Goethe University Frankfurt

University of Piraeus, September 2008

Motivation

- Basic **Lucas tree** equilibrium model (Lucas 1978)
  - only one asset ('the market portfolio') in positive net supply
  - representative agent
  - results (assuming log-utility and infinite horizon)
    - constant p/d ratio
    - no excess volatility
    - constant risk-free rate (term structure is flat)
- Extension by **Cochrane, Longstaff, Santa-Clara** (CLS 2008)
  - two trees, structurally identical
  - representative agent
  - results (again assuming log-utility and infinite horizon)
    - p/d ratio for individual trees depends on dividend share
    - return volatility differs from dividend volatility

Motivation

- This paper: **heterogeneous assets**
  - focus on information about assets
    - asset is characterized by expected dividend growth rate (drift) and uncertainty of dividend growth rate (volatility)
    - well-known asset: drift and volatility of dividend are known
    - unfamiliar asset: drift is stochastic and maybe unobservable, volatility is known
  - main question: resulting differences between stocks
- Furthermore: **heterogeneous agents**
  - focus on agents' information and learning process
    - irrational investors: use only information from dividend process
    - rational investors: use more information (signal) for learning
  - main questions
    - impact of information quality on equilibrium outcome
    - trading between investors, long-run survival

Results

- **Stochastic dividend drift ...**
  - return volatilities of both stocks increase
  - return correlations change from positive to negative

+ **Learning (stochastic drift is not observable) ...**
  - volatility and expected excess return
    - increase for unfamiliar asset
    - decrease for well-known asset

    effect is the larger the less precise the signal is
  - (mostly) negative return correlation between stocks

+ **Heterogeneous investors (rational vs. irrational) ...**
  - large trading volume in the unfamiliar stock (when small)
  - trading volume increases in difference in beliefs and information precision
  - irrational investors are driven out of the market rather fast

Model Setup: Economy

- Lucas tree economy with two independent **dividend** processes

$$\begin{aligned} \text{Blue chip:} \quad dB_t &= \mu_B B_t dt + \sigma_B B_t dW_{B,t} \\ \text{Young firm:} \quad dY_t &= \mu_{\mathbf{Y,t}} Y_t dt + \sigma_Y Y_t dW_{Y,t} \end{aligned}$$

- **Expected dividend growth rate** for Y is stochastic

$$d\mu_{Y,t} = \lambda \left(\bar{\mu}_Y - \mu_{Y,t} \right) dt + \sigma_\mu dW_{\mu,t}$$

  - uncertain dividend growth rate reflects 'young' firm with additional source of uncertainty
  - structural difference, not just more or less volatile dividends

- **Signal**: may contain information about dividend growth rate

$$\frac{ds_t}{s_t} = \alpha_{\mathbf{t}} dt + \sigma_s dW_{s,t}$$

- W_B, W_Y, W_μ, W_s: independent standard Brownian motions

Model Setup: Investors

- Investors: identical preferences . . .
  - derive utility from consumption
  - infinite planning horizon
  - log-utility, time-discount rate ρ

$$E\left[\int_0^\infty e^{-\rho t} \ln c_t \, dt\right]$$

- . . . but maybe different information

Model Setup: Heterogeneous Information

- In general: dividend drift $\mu_{Y,t}$ cannot be observed
- Investor j learns about dividend drift $\mu_{Y,t}$
  - current estimate: $\mu^j_{Y,t} := \mathbb{E}^j[\mu_{Y,t}|\mathcal{F}^j_t]$
  - variance of estimation error: $\gamma^j_t := \mathbb{E}^j[(\mu^j_{Y,t} - \mu_{Y,t})^2|\mathcal{F}^j_t]$
- Sources of information: observed dividend Y, maybe signal
- **Different priors about structure of model**
  - Irrational investors assume signal to be pure noise: $\alpha_t =$ noise
    $\longrightarrow$ base inference on observed dividend only
  - Rational investors assume signal to contain information: $\alpha_t = \mu_{Y,t}$
    $\longrightarrow$ base inference on observed dividend and signal
    limiting case: perfect signal with $\sigma_s = 0$
    $\longrightarrow$ dividend drift observable
- Even with the same information, investors do not agree ...
  - research reports in lifesciences $\longrightarrow$ different conclusions
  - recommendations of analysts $\longrightarrow$ differ from each other

Homogeneous Economy

- **Homogeneous economy:** benchmark case
  - all investors are equal
  - difference to CLS: stochastic Y-drift, learning
- Equilibrium interest rate in type j economy

$$r_t^j = \rho + z_t \mu_B + (1 - z_t)\mu_{Y,t}^j - z_t^2 \sigma_B^2 - (1 - z_t)^2 \sigma_Y^2$$

  - z: dividend share, $z_t = \frac{B_t}{B_t + Y_t}$
  - structurally equal in all homogeneous economies (CLS, II, RI)
- Market prices of risk

$$\theta_{B,t}^j = z_t \sigma_B \qquad \theta_{Y,t}^j = (1 - z_t)\sigma_Y$$
$$\theta_{s,t}^j = 0$$

  - identical in all homogeneous economies (CLS, II, RI)
  - signal shocks: not priced (no impact on consumption)

Homogeneous Economy

- Price of **market portfolio** with dividend $D = B + Y$:

$$P_{M,t} = \frac{D_t}{\rho}$$

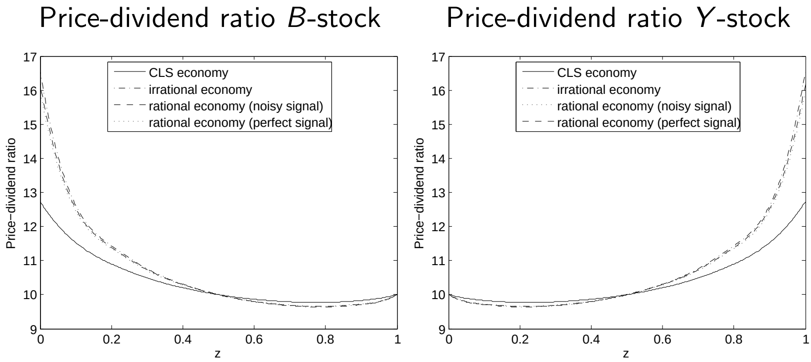
  - does not depend on uncertainty about dividend drift (due to log-utility, investor is myopic)

- Prices of **individual stocks**

$$P_{B,t} = (B_t + Y_t)\,\mathbb{E}_t^j\left[\int_0^\infty e^{-\rho\tau}\frac{B_{t+\tau}}{B_{t+\tau}+Y_{t+\tau}}d\tau\right]$$
$$P_{Y,t} = (B_t + Y_t)\,\mathbb{E}_t^j\left[\int_0^\infty e^{-\rho\tau}\frac{Y_{t+\tau}}{B_{t+\tau}+Y_{t+\tau}}d\tau\right]$$

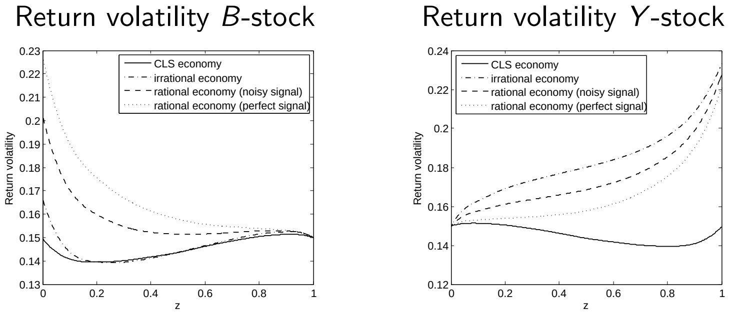
  - depend on type of economy
  - remark on calculation: prices calculated by Fourier inversion

Numerical Example: Price-Dividend Ratios



- Fundamental effect on p/d ratio (CLS): desire to diversify, increases price of stock with small dividend share
- **Stochastic drift** increases p/d ratio for stock with small dividend share further
- Impact of **information quality** is negligible
- Trees are symmetric: no differences between stocks

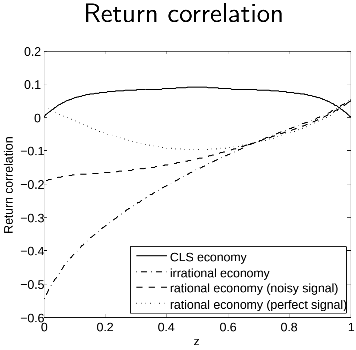
Numerical Example: Return Volatilities



- **Stochastic drift** increases volatilities of both assets
- **Learning** (Y-drift is not observable)
  - B-stock: lower volatility
    (smaller reaction to less precise signal, small exposure to Y-risk increases)
  - Y-stock: higher volatility
    (exposure to signal decreases, large exposure to Y-risk increases further)

effect is the larger the less precise the signal is

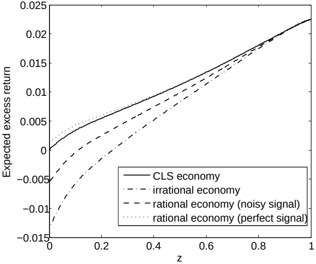
Numerical Example: Return Correlations



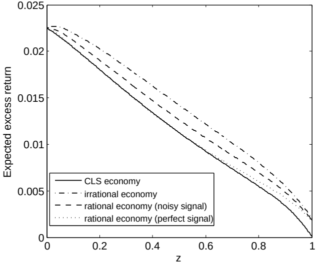
- **Stochastic drift**
  - $\mu_{Y,t}$ increases: Y-stock more attractive relative to B-stock
  - correlations change from positive (CLS) to negative
- **Learning** (Y-drift is not observable)
  - negative correlation increases in absolute terms (large Y)
  - effect is the large the less precise the signal is

Numerical Example: Risk Premia

Risk Premium *B*-stock



Risk Premium *Y*-stock



- Market prices of risk (structurally) equal in all four economies
- **Stochastic drift**: negligible impact
- **Learning** induces additional exposure to W_Y
  - B-stock: negative exposure → negative risk premium
  - Y-stock: large positive exposure → higher risk premium

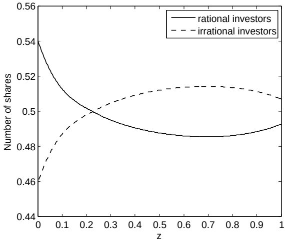
effect is the larger the less precise the signal is

Heterogeneous Economy

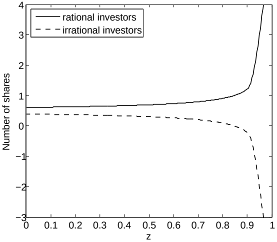
- **Heterogeneous Economy**
  - both rational and irrational investors (both have log-utility)
  - complete market
- Equilibrium interest rate
  - weighted average of interest rates in homogeneous economy
  - weights: consumption shares
- Market prices of risk
  - similar to homogeneous economy
  - market price of W_Y depends on relative optimism/pessimism
  - signal risk: not priced
- Asset prices
  - weighted averages
  - return volatilities, correlations, expected excess returns: between values in homogeneous economies in most cases

Optimal Portfolios (Scenario: RI More Optimistic)

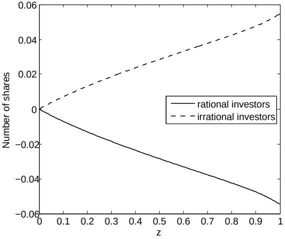
Number of B-shares



Number of Y-shares



'Pure signal risk' contracts

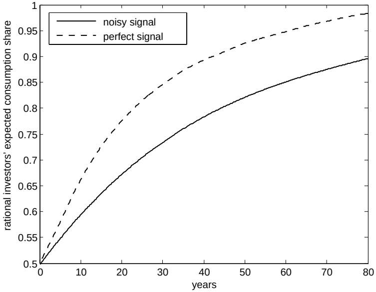


- Initial positions: equal shares of stocks, no 'signal-risk'-asset
- Investors agree on W_B risk: little trading volume in B-stock
- Positions in Y-stock depend on relative optimism/pessimism
  - optimistic investor (here: rational) holds larger position
  - high leverage when Y-stock is small
- Pure 'signal-risk'-asset eliminates exposure to signal risk
- Trading volume increases in differences in beliefs and in signal quality

Long-Run Survival

- Irrational investors will ultimately be driven out of the market (because they use an incorrect model structure)
- Important question: How long does it take?
  - previous papers: very long (several hundred years ...)
  - our paper: roughly 20-30 years until irrational investors have lost half of their initial wealth
- Reasons for fast natural selection
  - log-utility
  - modeling choice for irrationality

Expected wealth share of rational investors under true measure



Conclusion

- Economy with **heterogeneous assets** . . .
  - dividend drift of one asset is stochastic (but observable)
    - return volatilities of both assets increases
    - return correlation changes from positive to negative
  - investors learn about stochastic dividend drift
    - well-known asset: lower volatility, lower risk premium
    - unfamiliar asset: higher volatility, higher risk premium
    - large negative return correlation when unfamiliar asset is large
- . . . and **heterogeneous investors**
  - prices are weighted average of homogeneous economies
  - portfolio holdings depend on differences in beliefs and on information precision
  - moderate trading volume in well known stock, high trading volume in small unfamiliar stock
  - irrational investors driven out of the market rather fast