(Τιτλοποιήσεις)

❖ 26.11, 26.12, 26.13, 26.18, 26.19, 26.31, 26.41

26.11 *Consider a GNMA mortgage pool with principal of $20 million. The maturity is 30 years with a monthly mortgage payment of 10 percent per year. Assume no prepayments.*

a. *What is the monthly mortgage payment (100 percent amortizing) on the pool of mortgages?*

ANSWER: The monthly mortgage payment, PMT, is:
$20m = PVA_{n=360, k=0.8333} \times (PMT) => PMT = \$175,514.31$
(the monthly interest rate is 0.10/ 12 = 0.00833)

b. *If the GNMA insurance fee is 6 basis points and the servicing fee is 44 basis points, what is the yield on the GNMA pass-through?*

ANSWER: The GNMA's annual interest rate is 0.10 - 0.0044 - 0.0006 = 9.5 percent. The monthly interest rate is 0.095/12 = 0.0079167 or 0.79167 percent.

c. *What is the monthly payment on the GNMA in part (b)?*

ANSWER: The monthly GNMA payment, PMT, is: $20m = PVA_{n=360, k=0.79167\%} \times PMT =>$ PMT = $168,170.84

d. *Calculate the first monthly servicing fee paid to the originating FIs.*

ANSWER: The first monthly servicing fee, SF, is (the monthly fee rate is 0.44%/12 = 0.0367%):
$SF = (0.000367)\$20m = \$7,333.$

e. *Calculate the first monthly insurance fee paid to GNMA.*

ANSWER: The first monthly insurance payment, IP = (0.00005) $20m = $1,000
(The monthly insurance rate is 0.06%/12 = 0.005%)

26.12 *Calculate the value of (a) the mortgage pool and (b) the GNMA pass-through in question 11 if market interest rates increase 50 basis points. Assume no prepayments.*

ANSWER:
a. The mortgage pool's value, PV, is (the monthly discount rate is 10.5%/12 = 0.875%):
PV = $175,514.31 x $PVA_{n=360,k=0.875\%}$ = $19,187,359

b. The GNMA's value, PV, is (the monthly discount rate is 10%/12 = 0.8333%):
PV = $168,170.84 x $PVA_{n=360,k=0.8333\%}$ = $19,163,205.

26.13 *What would be the impact on GNMA pricing if the pass-through was not fully amortized? What is the present value of a $10 million pool of 15-year mortgages with an 8.5 percent per year monthly mortgage coupon if market rates are 5 percent? The GNMA guarantee fee is 6 basis points and the FI servicing fee is 44 basis points.*

a. *Assume that the GNMA is fully amortized.*

ANSWER: There are 180 monthly payments (15 years x 12 months). The GNMA monthly coupon rate is 8.5% - 0.5% = 8 percent per year, and the monthly GNMA pass-through payment is:
$10m = $PVA_{n=180, k=0.6667\%}$ x PMT => PMT = $95,565.21.

The present value of the GNMA at a 5 percent market rate is:
PV = $95,565.21 x $PVA_{n=180, k=0.004167\%}$ = $12,084,721.63.

b. *Assume that the GNMA is only half amortized. There is a lump sum payment at the maturity of the GNMA that equals 50 percent of the mortgage pool's face value.*

ANSWER: If there is a 50 percent amortization, the monthly GNMA pass-through payments are:
$10m = PMT x $PVA_{n=180, k=0.6667\%}$ + $5m x $PV_{n=180, k=0.6667\%}$ => PMT = $81,115.94

The present value of the GNMA at a 5 percent market rate is:
PV = $81,115.94 x $PVA_{n=180, k=0.004167\%}$ + $5m x $PV_{n=180, k=0.004167\%}$ = $12,623,051.35.

26.18. *If 150 $200,000 mortgages in a $60 million 15-year mortgage pool are expected to be prepaid in three years and the remaining 150 $200,000 mortgages are to be prepaid in four years, what is the weighted-average life of the mortgage pool? Mortgages are fully amortized, with mortgage coupon rates set at 10 percent to be paid annually.*

ANSWER: The annual mortgage payment is $60 million = $PVA_{n=15, k=10\%}$ x PMT => PMT = $7,888,426.61. Annual mortgage payments, with no prepayments, can be decomposed into principal and interest payments (in millions of $s):

Year	Balance	Payment	Interest Payment	Principal Payment	Remaining Principal
1	$60.000	$7.888	$6.000	$1.888	$58.112
2	58.112	7.888	5.811	2.077	56.034
3	56.034	7.888	5.603	2.285	53.749
4	53.749	7.888	5.375	2.513	51.236

The first year's interest is $6 million (0.10 x $60 million). Deducting this from the first year's mortgage payment yields a principal payment of $1,888,426.61 at the end of the first year, and an outstanding principal $58,111,573.39.

The second year's interest payment is 0.10 x $58,111,573.39 = $5,8111,157.34. Deducting this from the annual mortgage payment yields a second annual principal payment of $2,077,269.27, for a principal outstanding of $56,034,304.12.

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The third year's regular interest payment is $5.603 million. Deducting this from the annual mortgage payment yields a third annual principal payment of $2.285 million for a principal outstanding of $53,749,307.92.

The principal outstanding at the end of the fourth year, without prepayments, is $51,235,812.10. However, at the end of the third year, half of the mortgages in the mortgage pool are completely prepaid. That is, at the end of the third year, an additional principal payment of 50% x $53,749,307.92 = $26,874,653.96 is received for a remaining outstanding principal balance of $26.875 million. The total third year principal payment is therefore $29.16 million = the regular principal payment of $2.285 million plus an extra payment of $26.875 million.

The fourth year annual interest payment is 10% x $26.875 million = $2.687 million, leaving a regular fourth year principal payment of $7.888 million - $2.687 million = $5,200,961.21. This end-of-fourth-year principal payment would have left an outstanding principal balance of $21,673,692.75, which is paid in full at the end of the year. Fourth year principal payments total $26.875 million = $5.201 million, plus $21.674 million.

Prepayments alter the annual cash flows for years 3 and 4 as follows (in millions of $s):

Year	Balance	Payment	Interest	Principal	Balance
3	56.034	7.888	5.603	29.160	26.875
4	26.875	7.888	2.687	26.875	0

Calculating the weighted average life:

Time	Expected Principal Payments	Time x Principal
1	1.888m	1.888
2	2.077m	4.154
3	29.160m	87.48
4	26.875m	107.5
	60.000m	201.022

WAL = 201.022/60 = 3.35 years

26.19   *An FI originates a pool of 500 30-year mortgages, each averaging $150,000 with an annual mortgage coupon rate of 8 percent. Assume that the GNMA credit risk insurance fee is 6 basis points and that the FI's servicing fee is 19 basis points.*

a.   *What is the present value of the mortgage pool?*

  ANSWER: PV = 500 x $150,000 = $75 million

b.   *What is the monthly mortgage payment?*

  ANSWER: There are 360 monthly mortgage payments (30 years x 12 months). Monthly mortgage payments are $75,000,000 = PVA$_{n=360, k=0.6667\%}$ x PMT => PMT = $550,323.43.

c. *For the first two payments, what portion is interest and what portion is principal repayment?*

ANSWER: For the first monthly payment, the monthly interest is 0.08/12 x $75 million = $500,000. Therefore, for the first monthly mortgage payment, $50,323.43 is repayment of principal.

For the second monthly payment, the principal outstanding is $75m - $50,323.43 = $74,949,676.57. The monthly interest payment is $499,664.51. The principal payment in the second month is $550,323.43 - $499,664.51 = $50,658.92.

d. *What are the expected monthly cash flows to GNMA bondholders?*

ANSWER: The GNMA bond rate is 8% - (0.06% + 0.19%) = 7.75 percent. GNMA bondholders receive monthly payments of $75m = $PVA_{n=360, k=0.0775/12}$ x PMT => PMT = $537,309.18.

e. *What is the present value of the GNMA pass-through bonds? Assume that the risk- adjusted market annual rate of return is 8 percent compounded monthly.*

ANSWER: The discount yield is 8 percent annually, compounded monthly. The present value of the GNMA pass-through bonds is PV = $537,309.18 x $PVA_{n=360, k=0.6667\%}$ = $73,226,373.05.

f. *Would actual cash flows to GNMA bondholders deviate from expected cash flows as in part (d)? Why or why not?*

ANSWER: Actual payments will equal expected payments if and only if no prepayments are made. If any mortgages are prepaid as a result of refinancing or homeowner mobility, then the monthly payments will change. In the month in which prepayments are made, monthly payments will increase to reflect the principal repayments. In all subsequent months, monthly payments will decline to reflect the lower face value of the pass-through bonds.

g. *What are the expected monthly cash flows for the FI and GNMA?*

ANSWER: GNMA and the originating FI share the difference between the monthly mortgage payments and the GNMA pass-through payments $550,323.43 - $537,309.18 = $13,014.25. The originating bank gets 19 out of the 25 basis points (or 76 percent) for a payment of $9,890.83 monthly. GNMA receives the remaining 6 basis points (or 24 percent) for a payment of $3,123.42.

h. *If all of the mortgages in the pool are completely prepaid at the end of the second month, what is the pool's weighted-average life? Hint: Use your answer to part (c).*

ANSWER:

Time	Expected Principal Payments	Time x Principal
1 mo.	$50,323.43	$50,323.43
2 mo.	$74,949,676.57	$149,899,353.10
	$75,000,000.00	$149,949,676.53

WAL = (149,949,676.53/75 million) = 1.9993 months

The principal payment in the first month is $50,323.43. If the loan is paid off after month two, the principal payment in month two is $75 million - $50,323.43 = $74,949,676.57.

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i.   *What is the price of the GNMA pass-through security if its weighted-average life is equal to your solution for part (h)? Assume no change in market interest rates.*

ANSWER: The GNMA with a weighted average life of 1.9993 months has only two cash flows. The first month's cash flow is $537,309.18. The second month's cash flow is $537,309.18 plus the extra principal repayment of $74,899,017.65 = $75,436,326.83. The present value of the GNMA is PV = [$537,309.18/(1.006667)] + [$75,436,326.83/(1.006667)^2] = $74,974,229.44, where the monthly discount rate is 0.08/12.

j.   *What is the price of the GNMA pass-through with a weighted-average life equal to your solution for part (h) if market yields decline by 50 basis points?*

ANSWER: If market yields decline 50 basis points, to 7.5 percent compounded monthly, the present value of the GNMA is PV = [$537,309.18/(1.00625)] + [$75,436,326.83/(1.00625)^2] = $75,036,111.70, where the monthly discount rate is 0.075/12.

26.31.   *Consider $200 million of 30-year mortgages with a coupon of 10 percent per year paid quarterly.*

a.   *What is the quarterly mortgage payment?*

ANSWER: There are 120 quarterly payments over 30 years.   The quarterly mortgage payments are $200m = PVA$_{n=120, k=2.5\%}$ x PMT => PMT = $5,272,358.60.

b.   *What are the interest and principal repayments over the first year of life of the mortgages?*

ANSWER:

Quarter	Balance	(Fixed) Payment	Interest Payment	Principal Payment	Remaining Principal
1	$200,000,000	$5,272,359	$5,000,000	$272,359	$199,727,641
2	199,727,641	5,272,359	4,993,191	279,168	199,448,473
3	199,448,473	5,272,359	4,986,212	286,147	199,162,326
4	199,162,326	5,272,359	4,979,058	293,301	198,869,025

c.   *Construct a 30-year CMO using this mortgage pool as collateral. The pool has three tranches, where tranche A offers the least protection against prepayment and tranche C offers the most protection against prepayment. Tranche A of $50 million receives quarterly payments at 9 percent per year, tranche B of $100 million receives quarterly payments at 10 percent per year, and tranche C of $50 million receives quarterly payments at 11 percent per year.*

ANSWER:

	Tranche A	Tranche B	Tranche C	Total Issue
Principal amount	$50 million	$100 million	$50 million	$200 million
Interest rate	9 percent	10 percent	11 percent	10 percent
Quarterly interest on initial balance	$1,125,000	$2,500,000	$1,375,000	$5,000,000
Quarterly amortization				$5,272,359

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d. *Assume non-amortization of principal and no prepayments. What are the total promised coupon payments to the three classes? What are the principal payments to each of the three classes for the first year?*

ANSWER: Regular tranche A interest payments are $1.125 million quarterly. If there are no prepayments, then the regular GNMA quarterly payment of $5,272,359 is distributed among the three tranches. Five million is the total coupon interest payment for all three tranches. Therefore, $272,359 of principal is repaid each quarter. Tranche A receives all principal payments. Tranche A cash flows are $1,125,000 + $272,359 = $1,397,359 quarterly. The cash flows to tranches B and C are the scheduled interest payments.

Tranche A amortization schedule:

Quarter	Balance	Payment	Interest Payment	Principal Payment	Remaining Principal
1	$50,000,000	$1,397,359	$1,125,000	$272,359	$49,727,641
2	49,727,641	1,397,359	1,118,872	278,487	49,449,154
3	49,449,154	1,397,359	1,112,606	284,753	49,164,401
4	49,164,401	1,397,359	1,106,199	291,160	48,873,241

Tranche B amortization schedule:

Quarter	Balance	Payment	Interest Payment	Principal Payment	Remaining Principal
1	$100.000m	$2.500m	$2.500m	$0.0m	$100.000m
2	100.000m	2.500m	2.500m	0.0m	100.000m
3	100.000m	2.500m	2.500m	0.0m	100.000m
4	100.000m	2.500m	2.500m	0.0m	100.000m

Tranche C amortization schedule:

Quarter	Balance	Payment	Interest Payment	Principal Payment	Remaining Principal
1	$50.000m	$1.375m	$1.375m	$0.0m	$50.000m
2	50.000m	1.375m	1.375m	0.0m	50.000m
3	50.000m	1.375m	1.375m	0.0m	50.000m
4	50.000m	1.375m	1.375m	0.0m	50.000m

e. *If, over the first year, the trustee receives quarterly prepayments of $10 million on the mortgage pool, how are these funds distributed?*

ANSWER: The quarterly prepayments of $10 million will be credited entirely to tranche A until tranche A is completely retired. Then prepayments will be paid entirely to tranche B. The amortization schedule for tranche A for the first year is shown below. This amortization schedule assumes that the trustee has a quarterly payment amount from the mortgage pool of $5,272,359.

Quarter	Balance	Payment	Interest Payment	Principal Payment	Remaining Principal
1	$50,000,000	$11,397,359	$1,125,000	$10,272,359	$39,727,641
2	39,727,641	11,397,359	893,872	10,503,487	29,224,154
3	29,224,154	11,397,359	657,543	10,739,816	18,484,338
4	18,484,228	11,397,359	415,898	10,981,461	7,502,877

However, since some of the mortgages will be paid off early, the actual payment received by the trustee from the mortgage pool will decrease each quarter. Thus, the payment for the second quarter will decrease from $5,272,359 to $5,008,381 (n = 119 quarters, i = 10 percent, mortgage principal = $189,727,641). The CMO amortization schedule for tranche A given that the mortgage payments decrease with the prepayments is given below. The revised mortgage payment for each quarter is shown in the last column.

Quarter	Balance	Payment	Interest Payment	Principal Payment	Remaining Principal	Mortgage Payment
1	$50,000,000	$11,397,359	$1,125,000	$10,272,359	$39,727,641	$5,272,359
2	39,727,641	11,133,381	893,872	10,239,509	29,488,132	5,008,381
3	29,488,132	10,869,713	663,483	10,206,230	19,281,902	4,744,713
4	19,281,902	10,606,344	433,843	10,172,501	9,109,401	4,481,344

f.   *How are the cash flows distributed if prepayments in the first half of the second year are $20 million quarterly?*

ANSWER: The amortization schedules for tranches A and B are shown below. Again the mortgage payments from the mortgage holders are assumed to decrease as the prepayments occur.

Amortization schedule for tranche A:

Quarter	Balance	Tranche Payment	Interest Payment	Principal Payment	Remaining Principal	Mortgage Payment
5	$9,109,401	$20,342,263	$204,961	$20,137,301	-$11,027,900	$4,218,263

Amortization schedule for tranche B:

Quarter	Balance	Tranche Payment	Interest Payment	Principal Payment	Remaining Principal	Mortgage Payment
5	$100,000,000	$13,527,900	$2,500,000	$11,027,900	$88,972,100	$4,218,263
6	88,972,100	23,689,967	2,224,302	21,465,665	67,506,435	3,689,967

g.   *How can the CMO issuer earn a positive spread on the CMO?*

ANSWER: The way the terms of the CMO are structured, the average coupon rate on the three classes equals the mortgage coupon rate on the underlying mortgage pool. However, given the more desirable cash flow characteristics of the individual classes, the FI may be able to issue the CMO classes at lower coupon rates. The difference between the sum of all coupon payments promised on all CMO tranches and the mortgage coupon rate on the underlying mortgage pool is the FI's servicing fee.

26.41. *An FI originates a pool of real estate loans worth $20 million with maturities of 10 years and paying interest rates of 9 percent per year.*

a. *What is the average payment received by the FI, including both principal and interest, if no prepayment is expected over the life of the loan?*

ANSWER: The average payment is $20,000,000 = $PVA_{n=10,\ k=9\%}$ x PMT => PMT = $3,116,401.80

b. *If the loans are converted into pass-through certificates and the FI charges a servicing of 50 basis points, including insurance, what is the payment amount expected by the holders of the pass-through securities if no prepayment is expected?*

ANSWER: $20,000,000 = $PVA_{n=10,k=8.5\%}$ x PMT => PMT = $3,048,154.10

c. *Assume that the payments are separated into interest only (IO) and principal only (PO) payments, that prepayments of 5 percent occur at the end of years 3 and 4, and that the payment of the remaining principal occurs at the end of year 5. What are the expected annual payments for each instrument? Assume discount rates of 9 percent.*

ANSWER:

Year	Balance	Payment	Interest Payment	Principal Payment	Remaining Principal
1	$20,000,000.00	$3,116,401.80	$1,800,000.00	$1,316,401.80	$18,683,598.20
2	18,683,598.20	$3,116,401.80	$1,681,523.84	$1,434,877.96	$17,248,720.24
3	17,248,720.24	$3,978,837.81	$1,552,384.82	$2,426,452.99	$14,822,267.25
4	14,822,267.25	$3,857,515.16	$1,334,004.05	$2,523,511.11	$12,298,756.14
5	12,298,756.14	$13,405,644.19	$1,106,888.05	$12,298,756.14	$0.00

d. *What is the market value of IOs and POs if the market interest rates for instruments of similar risk decline to 8 percent?*

ANSWER: The market value of the IO is found by discounting the interest payment column in part (c) at 8 percent. The PV = $6,074,497.66.

The market value of the PO is found by discounting the principal payment column in part (c) at 8 percent. The PV = $14,600,446.52.

Note that the PV of the total payments is $20,674,944.18 which is the sum of the PV of the IO and the PV of the PO.