

ΑΣΚΗΣΕΙΣ – Κεφάλαιο 3

(Πιστωτικός Κίνδυνος)

- ❖ 10.20, 10.23, 10.25, 10.26, 10.27, 10.34, 10.35, 10.38,
- ❖ 22.38, 22.40,
- ❖ 24.9, 24.12, 24.15, 24.16, 24.20, 24.21, 24.22

10.20 Suppose the estimated linear probability model used by an FI to predict business loan applicant default probabilities is $PD = 0.03X_1 + 0.02X_2 - 0.05X_3 + \text{error}$, where X_1 is the borrower's debt/equity ratio, X_2 is the volatility of borrower earnings, and $X_3 = 0.10$ is the borrower's profit ratio. For a particular loan applicant, $X_1 = 0.75$, $X_2 = 0.25$, and $X_3 = 0.10$.

a. What is the projected probability of default for the borrower?

ANSWER: $PD = 0.03(0.75) + 0.02(0.25) - 0.05(0.10) = 0.0225$

b. What is the projected probability of repayment if the debt/equity ratio is 2.5?

ANSWER: $PD = 0.03(2.5) + 0.02(0.25) - 0.05(0.10) = 0.075$
The expected probability of repayment is $1 - 0.075 = 0.925$.

c. What is a major weakness of the linear probability model?

ANSWER: A major weakness of this model is that the estimated probabilities can be below 0 percent or above 100 percent, an occurrence that does not make economic or statistical sense.

10.23 MNO Inc., a publicly traded manufacturing firm in the United States, has provided the following financial information in its application for a loan. All numbers are in thousands of dollars.

| <u>Assets</u> | | <u>Liabilities and Equity</u> | |
|----------------------|--------------|-------------------------------|--------------|
| Cash | \$ 20 | Accounts payable | \$ 30 |
| Accounts receivables | 90 | Notes payable | 90 |
| Inventory | 90 | Accruals | 30 |
| | | Long-term debt | 150 |
| Plant and equipment | <u>500</u> | Equity (ret. earnings = \$22) | <u>400</u> |
| Total assets | <u>\$700</u> | Total liabilities and equity | <u>\$700</u> |

Also assume sales = \$500,000 ; cost of goods sold = \$360,000; and the market value of equity is equal to the book value.

- a. *What is the Altman discriminant function value for MNO Inc.? Recall that:
 Net working capital = Current assets - Current liabilities.
 Current assets = Cash + Accounts receivable + Inventories.
 Current liabilities = Accounts payable + Accruals + Notes payable.
 EBIT = Revenues - Cost of goods sold.*

ANSWER: Altman's discriminant function is given by: $Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5$
 All numbers are in \$000s.

$$\begin{aligned} X_1 &= (20 + 90 + 90 - 30 - 90 - 30) / 700 = 0.0714 & X_1 &= \text{Working capital/total assets (TA)} \\ X_2 &= (22) / 700 = 0.0314 & X_2 &= \text{Retained earnings/TA} \\ X_3 &= (500 - 360) / 700 = 0.20 & X_3 &= \text{EBIT/TA} \\ X_4 &= 400 / 150 = 2.6667 & X_4 &= \text{Market value of equity/Book value of long-term debt} \\ X_5 &= 500 / 700 = 0.7143 & X_5 &= \text{Sales/TA} \end{aligned}$$

$$\begin{aligned} Z &= 1.2(0.0714) + 1.4(0.0314) + 3.3(0.20) + 0.6(2.6667) + 1.0(0.7143) = 3.104 \\ &= 0.0857 + 0.0440 + 0.6600 + 1.600 + 0.7143 = 3.104 \end{aligned}$$

- b. *Based on the Altman's Z-score only, should you approve MNO Inc.'s application to your bank for a \$500,000 capital expansion loan?*

ANSWER: Since the Z-score of 3.104 is greater than 2.99, ABC Inc.'s application for a capital expansion loan should be approved.

- c. *If sales for MNO were \$300,000, the market value of equity was only half of book value, and all other values are unchanged, would your credit decision change?*

ANSWER: ABC's EBIT would be \$300,000 - \$360,000 = -\$60,000.

$$\begin{aligned} X_1 &= (20 + 90 + 90 - 30 - 90 - 30) / 700 = 0.0714 \\ X_2 &= 22 / 700 = 0.0314 \\ X_3 &= -60 / 700 = -0.0857 \\ X_4 &= 200 / 150 = 1.3333 \\ X_5 &= 300 / 700 = 0.4286 \end{aligned}$$

$$Z = 1.2(0.0714) + 1.4(0.0314) + 3.3(-0.0857) + 0.6(1.3333) + 1.0(0.4286) = 1.0754$$

Since ABC's Z-score falls to $1.0754 < 1.81$, credit should be denied.

- d. *Would the discriminant function change for firms in different industries? Would the function be different for manufacturing firms in different geographic sections of the country? What are the implications for the use of these types of models by FIs?*

ANSWER: Discriminant function models are very sensitive to the weights for the different variables. Since different industries have different operating characteristics, a reasonable answer would be yes with the condition that there is no reason that the functions could not be similar for different industries. In the retail market, the demographics of the market play a big role in the value of the weights. For example, credit card companies often evaluate different models for different areas of the country. Because of the sensitivity of the models, extreme care should be taken in the process of selecting the correct sample to validate the model for use.

10.25. *If the rate on one-year Treasury strips currently is 6 percent, what is the repayment probability for each of the following two securities? Assume that if the loan is defaulted, no payments are expected. What is the market-determined risk premium for the corresponding probability of default for each security?*

- a. *One-year AA-rated zero coupon bond yielding 9.5 percent.*

ANSWER: Probability of repayment $= p = (1 + i)/(1 + k)$
 For an AA-rated bond $= (1 + 0.06)/(1 + 0.095) = 0.968$, or 96.80 percent
 \Rightarrow probability of default $= 1 - 0.968 = 0.032$, or 3.20%

The market determined risk premium is $0.095 - 0.060 = 0.035$ or 3.5 percent. This implies a probability of default of 3.20 percent on a corporate bond requires an FI to set a risk premium of 3.5 percent.

- b. *One-year BB-rated zero coupon bond yielding 13.5 percent.*

ANSWER: Probability of repayment $= p = (1 + i)/(1 + k)$
 For BB-rated bond $= (1 + 0.06)/(1 + 0.135) = 0.9339$ percent
 \Rightarrow probability of default $= 1 - 0.9339 = 0.0661$, or 6.61%

The market determined risk premium is $0.135 - 0.060 = 0.075$ or 7.50 percent. This implies a probability of default of 6.61 percent on a corporate bond requires an FI to set a risk premium of 7.5 percent.

10.26. *A bank has made a loan charging a base lending rate of 10 percent. It expects a probability of default of 5 percent. If the loan is defaulted, the bank expects to recover 50 percent of its money through the sale of its collateral. What is the expected return on this loan?*

ANSWER: $E(r) = p(1 + k) + (1 - p)(1 + k)(\gamma)$ where γ is the percentage generated when the loan is defaulted. $E(r) = 0.95(1 + 0.10) + 0.05(1 + 0.10)(0.50) = 1.0450 + 0.0275 = 1.0725 - 1.0 = 7.25\%$

10.27 Assume a one-year Treasury strip is currently yielding 5.5 percent and an AAA-rated discount bond with similar maturity is yielding 8.5 percent.

- a. If the expected recovery from collateral in the event of default is 50 percent of principal and interest, what is the probability of repayment of the AAA-rated bond? What is the probability of default?

ANSWER: $p(1+k) + \gamma(1-p)(1+k) = 1+i$. Solve for the probability of repayment (p):

$$p = \frac{\frac{1+i}{1+k} - \gamma}{1-\gamma} = \frac{\frac{1.055}{1.085} - 0.5}{1-0.5} = 0.9447 \text{ or } 94.47 \text{ percent}$$

Therefore the probability of default is $1.0 - 0.9447 = 0.0553$ or 5.53 percent.

- b. What is the probability of repayment of the AAA-rated bond if the expected recovery from collateral in the case of default is 94.47 percent of principal and interest? What is the probability of default?

ANSWER: $p = \frac{\frac{1+i}{1+k} - \gamma}{1-\gamma} = \frac{\frac{1.055}{1.085} - 0.9447}{1-0.9447} = 0.5000 \text{ or } 50.00 \text{ percent}$

Therefore the probability of default is $1.0 - 0.5000 = 0.5000$ or 50.00 percent.

- c. What is the relationship between the probability of default and the proportion of principal and interest that may be recovered in the case of default on the loan?

ANSWER: The proportion of the loan's principal and interest that is collectible on default is a perfect substitute for the probability of repayment should such defaults occur.

10.34. The following is a schedule of historical defaults (yearly and cumulative) experienced by an FI manager on a portfolio of commercial and mortgage loans.

| <u>Loan Type</u> | <u>Years after Issuance</u> | | | | |
|--------------------|-----------------------------|----------------|----------------|----------------|----------------|
| | <u>1 Year</u> | <u>2 Years</u> | <u>3 Years</u> | <u>4 Years</u> | <u>5 Years</u> |
| <i>Commercial:</i> | | | | | |
| Annual default | 0.00% | | 0.50% | | 0.30% |
| Cumulative default | | 0.10% | | 0.80% | |
| <i>Mortgage:</i> | | | | | |
| Annual default | 0.10% | 0.25% | 0.60% | | 0.80% |
| Cumulative default | | | | 1.64% | |

a. Complete the blank spaces in the table.

ANSWER:

| | |
|----------------------------|--|
| Commercial: Annual default | 0.00%, <u>0.10%</u> , 0.50%, <u>0.20%</u> , and 0.30% |
| Cumulative default: | <u>0.00%</u> , 0.10%, <u>0.60%</u> , 0.80%, and <u>1.10%</u> |
| Mortgage: Annual default | 0.10%, 0.25%, 0.60%, <u>0.70%</u> , and 0.80% |
| Cumulative default | <u>0.10%</u> , <u>0.35%</u> , <u>0.95%</u> , 1.64%, and <u>2.43%</u> |

Note: The annual survival rate is $p_t = 1 - \text{annual default rate}$, and the cumulative default rate for $n = 3$ of mortgages is $1 - (p_1 \times p_2 \times p_3 \times p_4) = 1 - (1 - 0.001) \times (1 - 0.0025) \times (1 - 0.006) = 1 - (0.999 \times 0.9975 \times 0.9940) = 0.0164 = 1.64\%$

b. What are the probabilities that each type of loan will not be in default after 5 years?

ANSWER: The cumulative survival rate is $= (1 - \text{MMR}_1) \times (1 - \text{MMR}_2) \times (1 - \text{MMR}_3) \times (1 - \text{MMR}_4) \times (1 - \text{MMR}_5)$ where MMR = marginal mortality rate

Commercial loan $= (1 - 0.00) \times (1 - 0.001) \times (1 - 0.005) \times (1 - 0.002) \times (1 - 0.003) = 0.989$ or 98.9%.

Mortgage loan $= (1 - 0.001) \times (1 - 0.0025) \times (1 - 0.006) \times (1 - 0.007) \times (1 - 0.008) = 0.9757$ or 97.57%.

c. What is the measured difference between the cumulative default (mortality) rates for commercial and mortgage loans after four years?

ANSWER:

Looking at the table, the cumulative rates of default in year 4 are 0.80% and 1.64%, respectively, for the commercial and mortgage loans. Another way of estimation is:

Cumulative mortality rate (CMR) $= 1 - (1 - \text{MMR}_1)(1 - \text{MMR}_2)(1 - \text{MMR}_3)(1 - \text{MMR}_4)$
 For commercial loan $= 1 - (1 - 0.000)(1 - 0.0010)(1 - 0.0050)(1 - 0.0020)$
 $= 1 - 0.9920 = 0.0080$ or 0.80 percent.

For mortgage loan $= 1 - (1 - 0.0010)(1 - 0.0025)(1 - 0.0060)(1 - 0.0070)$
 $= 1 - 0.98359 = 0.01641$ or 1.64 percent.

The difference in cumulative default rates is $1.64 - 0.80 = 0.84$ percent.

10.35. The table below shows the dollar amounts of outstanding bonds and corresponding default amounts for every year over the past five years. Note that the default figures are in millions, while those outstanding are in billions. The outstanding figures reflect default amounts and bond redemptions.

| <u>Loan Type</u> | <u>Years after Issuance</u> | | | | |
|------------------------------------|-----------------------------|----------------|----------------|----------------|----------------|
| | <u>1 Year</u> | <u>2 Years</u> | <u>3 Years</u> | <u>4 Years</u> | <u>5 Years</u> |
| A-rated: Annual default (millions) | 0 | 0 | 0 | \$ 1 | \$ 2 |
| Outstanding (billions) | \$100 | \$95 | \$93 | \$91 | \$88 |
| B-rated: Annual default (millions) | 0 | \$ 1 | \$ 2 | \$ 3 | \$ 4 |
| Outstanding (billions) | \$100 | \$94 | \$92 | \$89 | \$85 |
| C-rated: Annual default (millions) | \$ 1 | \$ 3 | \$ 5 | \$ 5 | \$ 6 |
| Outstanding (billions) | \$100 | \$97 | \$90 | \$85 | \$79 |

a. What are the annual and cumulative default rates of the above bonds?

ANSWER:

A-rated Bonds

| <u>Year</u> | <u>Millions Default</u> | <u>Millions Balance</u> | <u>Annual Default</u> | <u>Survival = 1 - An. Def.</u> | <u>Cumulative Default Rate</u> | <u>% Cumulative Default Rate</u> |
|-------------|-------------------------|-------------------------|-----------------------|--------------------------------|--------------------------------|----------------------------------|
| 1 | 0 | 100,000 | 0.000000 | 1.000000 | 0.000000 | 0.0000% |
| 2 | 0 | 95,000 | 0.000000 | 1.000000 | 0.000000 | 0.0000% |
| 3 | 0 | 93,000 | 0.000000 | 1.000000 | 0.000000 | 0.0000% |
| 4 | 1 | 91,000 | 0.000011 | 0.999989 | 0.000011 | 0.0011% |
| 5 | 2 | 88,000 | 0.000023 | 0.999977 | 0.000034 | 0.0034% |

Where cumulative default for nth year = 1 - product of survival rates to that year.

B-rated Bonds

| <u>Year</u> | <u>Millions Default</u> | <u>Millions Balance</u> | <u>Annual Default</u> | <u>Survival = 1 - An. Def.</u> | <u>Cumulative Default Rate</u> | <u>% Cumulative Default Rate</u> |
|-------------|-------------------------|-------------------------|-----------------------|--------------------------------|--------------------------------|----------------------------------|
| 1 | 0 | 100,000 | 0.000000 | 1.000000 | 0.000000 | 0.0000% |
| 2 | 1 | 94,000 | 0.000011 | 0.999989 | 0.000011 | 0.0011% |
| 3 | 2 | 92,000 | 0.000022 | 0.999978 | 0.000032 | 0.0032% |
| 4 | 3 | 89,000 | 0.000034 | 0.999966 | 0.000066 | 0.0066% |
| 5 | 4 | 85,000 | 0.000047 | 0.999953 | 0.000113 | 0.0113% |

C-rated Bonds

| <u>Year</u> | <u>Millions Default</u> | <u>Millions Balance</u> | <u>Annual Default</u> | <u>Survival = 1 - An. Def.</u> | <u>Cumulative Default Rate</u> | <u>% Cumulative Default Rate</u> |
|-------------|-------------------------|-------------------------|-----------------------|--------------------------------|--------------------------------|----------------------------------|
| 1 | 1 | 100,000 | 0.000010 | 0.999990 | 0.000010 | 0.0010% |
| 2 | 3 | 97,000 | 0.000031 | 0.999969 | 0.000041 | 0.0041% |
| 3 | 5 | 90,000 | 0.000056 | 0.999944 | 0.000096 | 0.0096% |
| 4 | 5 | 85,000 | 0.000059 | 0.999941 | 0.000155 | 0.0155% |
| 5 | 6 | 79,000 | 0.000076 | 0.999924 | 0.000231 | 0.0231% |

| Bond Type | Years after Issuance | | | | |
|-------------------------|----------------------|---------|---------|---------|---------|
| | 1 Year | 2 Years | 3 Years | 4 Years | 5 Years |
| A-rated: Annual default | 0% | 0% | 0% | 0.0011% | 0.0023% |
| Cumulative default | 0% | 0% | 0% | 0.0011% | 0.0034% |
| B-rated: Annual default | 0% | 0.0011% | 0.0022% | 0.0034% | 0.0047% |
| Cumulative default | 0% | 0.0011% | 0.0032% | 0.0066% | 0.0113% |
| C-rated: Annual default | 0.0010 % | 0.0031% | 0.0056% | 0.0059% | 0.0076% |
| Cumulative default | 0.0010 % | 0.0041% | 0.0096% | 0.0155% | 0.0231% |

Note: These percentage values seem very small. More reasonable values can be obtained by increasing the default dollar values by a factor of ten, or by decreasing the outstanding balance values by a factor of 0.10. Either case will give the same answers that are shown below. While the percentage numbers seem somewhat more reasonable, the true values of the problem are (a) that default rates are higher on lower rated assets, and (b) that the cumulative default rate involves more than the sum of the annual default rates.

| C-rated Bonds | | Test with 10x default. | | | | |
|---------------|---------------------|------------------------|-------------------|----------------------------|----------------------------|------------------------------|
| Year | Millions Default | Millions Balance | Annual Default | Survival = 1 - An. Def. | Cumulative Default Rate | % Cumulative Default Rate |
| 1 | 10 | 100,000 | 0.000100 | 0.999900 | 0.000100 | 0.0100% |
| 2 | 30 | 97,000 | 0.000309 | 0.999691 | 0.000409 | 0.0409% |
| 3 | 50 | 90,000 | 0.000556 | 0.999444 | 0.000965 | 0.0965% |
| 4 | 50 | 85,000 | 0.000588 | 0.999412 | 0.001552 | 0.1552% |
| 5 | 60 | 79,000 | 0.000759 | 0.999241 | 0.002311 | 0.2311% |

More meaningful to use 0.10 x balance, will get same result.

10.38. A bank is planning to make a loan of \$5,000,000 to a firm in the steel industry. It expects to charge a servicing fee of 50 basis points. The loan has a maturity of 8 years with a duration of 7.5 years. The cost of funds (the RAROC benchmark) for the bank is 10 percent. The bank has estimated the maximum change in the risk premium on the steel manufacturing sector to be approximately 4.2 percent, based on two years of historical data. The current market interest rate for loans in this sector is 12 percent.

a. Using the RAROC model, determine whether the bank should make the loan?

ANSWER: RAROC = Fees and interest earned on loan/Loan or capital risk

Loan risk, or $\Delta LN = -D_{LN} \times LN \times (\Delta R / (1 + R)) = -7.5 \times \$5m \times (0.042 / 1.12) = -\$1,406,250$

Expected interest = $0.12 \times \$5,000,000 = \$600,000$

Servicing fees = $0.0050 \times \$5,000,000 = \$25,000$

Less cost of funds = $0.10 \times \$5,000,000 = -\$500,000$

Net interest and fee income = $\$125,000$

RAROC = $\$125,000 / 1,406,250 = 8.89$ percent. Since RAROC is lower than the cost of funds to the bank, the bank should not make the loan.

b. What should be the duration in order for this loan to be approved?

ANSWER: For RAROC to be 10 percent, loan risk should be:
 $\$125,000/\Delta LN = 0.10 \Rightarrow \Delta LN = 125,000 / 0.10 = \$1,250,000$
 $\Rightarrow -D_{LN} \times LN \times (\Delta R/(1 + R)) = 1,250,000$

$$D_{LN} = 1,250,000 / (5,000,000 \times (0.042/1.12)) = 6.67 \text{ years.}$$

Thus, this loan can be made if the duration is reduced to 6.67 years from 7.5 years.

c. Assuming that duration cannot be changed, how much additional interest and fee income will be necessary to make the loan acceptable?

ANSWER: Necessary RAROC = Income/Risk \Rightarrow Income = RAROC x Risk
 $= \$1,406,250 \times 0.10 = \$140,625$
 Therefore, additional income = $\$140,625 - \$125,000 = \$15,625$, or
 $\$15,625/\$5,000,000 = 0.003125 = 0.3125\%$.

Thus, this loan can be made if fees are increased from 50 basis points to 81.25 basis points.

d. Given the proposed income stream and the negotiated duration, what adjustment in the loan rate would be necessary to make the loan acceptable?

ANSWER: Need an additional $\$15,625 \Rightarrow \$15,625/\$5,000,000 = 0.003125$ or 0.3125%

| | | |
|-----------------------------|--------------------------|---------------------|
| Expected interest | = 0.123125 x \$5,000,000 | = \$615,625 |
| Servicing fees | = 0.0050 x \$5,000,000 | = \$25,000 |
| Less cost of funds | = 0.10 x \$5,000,000 | = <u>-\$500,000</u> |
| Net interest and fee income | | = <u>\$140,625</u> |

RAROC = $\$140,625/\$1,406,250 = 10.00$ percent = cost of funds to the bank.

Thus, increasing the loan rate from 12% to 12.3125% will make the loan acceptable.

22.38. *What is the gain on the purchase of a \$20 million credit forward contract with a modified duration of seven years if the credit spread between a benchmark Treasury bond and a borrowing firm's debt decreases by 50 basis points?*

ANSWER: The gain would be $(\Phi_T - \Phi_F) \times MD \times \$20 \text{ million} = 0.005 \times 7 \times \$20 \text{ million} = \$700,000$.

22.40. *A property-casualty (PC) insurance company has purchased catastrophe futures contracts to hedge against losses during the hurricane season. At the time of purchase, the market expected a loss ratio of 0.75. After processing claims from a severe hurricane, the PC actually incurred a loss ratio of 1.35. What amount of profit did the PC make on each \$25,000 futures contract?*

ANSWER: The payoff = actual loss ratio \times \$25,000 = 1.35 \times \$25,000 = \$33,750.

24.9 Two multinational FIs enter their respective debt markets to issue \$100 million of two-year notes. FI A can borrow at a fixed annual rate of 11 percent or a floating rate of LIBOR plus 50 basis points, repriced at the end of the year. FI B can borrow at a fixed annual rate of 10 percent or a floating rate of LIBOR, repriced at the end of the year.

a. If FI A is a positive duration gap insurance company and FI B is a money market mutual fund, in what market(s) should each firm borrow so as to reduce its interest rate risk exposure?

ANSWER: FI A will prefer to borrow in the fixed-rate debt market in order to generate positive cash flows when interest rates increase. This will offset the impact of an increase in interest rates, which would cause the market value of the insurance company's equity to decline. FI B will prefer to borrow in the floating rate debt market so as to better match the duration of its short-term assets.

b. In which debt market does FI A have a comparative advantage over FI B?

ANSWER: The matrix of possible interest rates is given below.

| | Fixed rate | Variable rate |
|------------|---------------|------------------|
| FI A | 11.0% | LIBOR+0.5% |
| FI B | 10.0% | LIBOR % |
| Difference | 1.0% | 0.5% |

FI A has a comparative advantage in the floating-rate market and FI B has a comparative advantage in the fixed-rate market. This is because the default risk premium of FI A over FI B is 50 basis points in the floating-rate market and 100 basis points in the fixed-rate market.

c. Although FI A is riskier than FI B and therefore must pay a higher rate in both the fixed-rate and floating-rate markets, there are possible gains to trade. Set up a swap to exploit FI A's comparative advantage over FI B. What are the total gains from the swap? Assume a swap intermediary fee of 10 basis points.

ANSWER: The total gains to the swap are 50 basis points (the price differential on FI A's default risk premium over FI B) less 10 basis points (the swap intermediary fee). Both FI A and B can exploit this price differential by issuing debt in the debt market in which they have comparative advantage and then swapping the interest payments. The 40 basis points can be allocated to either FI A and/or FI B according to the terms of the swap. A possible set of feasible swap rates that give all of the gains to FI A (see part (d) below) is illustrated here.

| FI A | Swap Cash Flows | | FI B |
|--|-------------------------------|---------|------------------------------|
| Fixed-rate assets | Fixed-rate swap payments | → 10.0% | Variable-rate assets |
| | LIBOR % | | |
| | ← Variable-rate swap payments | | |
| | Cash | | |
| Variable-rate liabilities @ LIBOR+0.5% | ← Financing Markets | → | Fixed-rate liabilities @ 10% |

Evidence that FI A receives all of the benefits is given in the payoff matrix below.

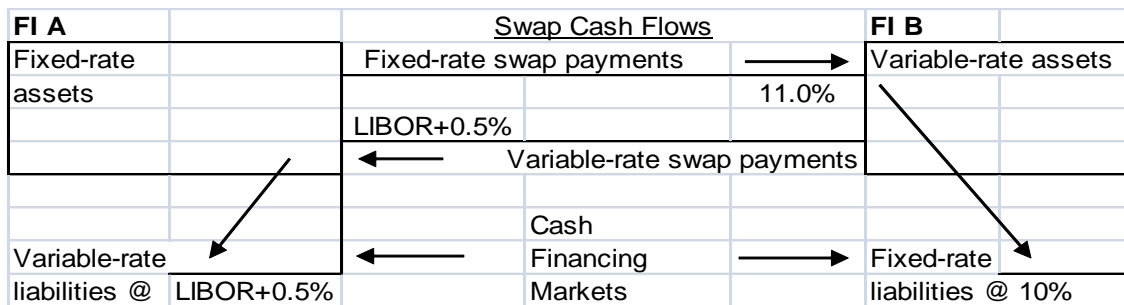
| FI A | FI B | |
|--------------------------------|------------|-------------------|
| Cash market liability rate | LIBOR+0.5% | 10.0% |
| Minus swap rate | -(LIBOR %) | -10.0% |
| Plus swap rate | + 10.0% | <u>+(LIBOR %)</u> |
| Net financing cost rate | 10.5% | LIBOR % |
| Less intermediary fee | 0.1% | |
| Financing cost rate net of fee | 10.6% | |

FI A is paying the intermediary fee, since FI B is receiving no benefits from this swap transaction. The 40 basis point net differential could be shared in a number of other combinations where FI A received most (exploited) of the benefit.

- d. *The gains from the swap can be apportioned between FI A and FI B through negotiation. What terms of swap would give all the gains to FI A? What terms of swap would give all the gains to FI B?*

ANSWER: All the gains go to FI A if FI B pays LIBOR for FI A's floating rate debt. Then FI A must pay 10 percent for FI B's fixed-rate debt plus 50 basis points on FI A's floating rate debt plus 10 basis points for the swap intermediary's fee. The total fixed annual interest cost to FI A is 10.6 percent, a savings of 40 basis points over the cash-market fixed rate of 11 percent. This swap rate apportionment is illustrated in part (c) above.

All the gains go to FI B if FI A pays 11 percent for FI B's fixed-rate, 10 percent debt. Then FI B pays LIBOR plus 50 basis points on FI A's floating rate debt for a net savings of 50 basis points. The savings occurs because FI B receives an excess 1 percent from FI A, but must pay 50 basis points more to FI A than it would pay in the cash floating-rate market. FI A must pay 11 percent against FI B's fixed-rate debt, but receives its exact liability payment from FI B. A diagram of this allocation is given below.



In this example, FI B would pay the swap intermediary fee of 10 basis points, and thus would realize a net, after-fee savings of 40 basis points. The payoff matrix is given below.

| | FI A | FI B |
|--------------------------------|----------------|-----------------------|
| Cash market liability rate | LIBOR+0.5% | 10.0% |
| Minus swap rate | -(LIBOR+0.5 %) | -11.0% |
| Plus swap rate | + 11.0% | <u>+(LIBOR+0.5 %)</u> |
| Net financing cost rate | 11.0% | LIBOR-0.5% |
| Less intermediary fee | | <u>0.1%</u> |
| Financing cost rate net of fee | | LIBOR-0.4% |

- e. Assume swap pricing that allocates all gains from the swap to FI A. If FI A buys the swap from FI B and pays the swap intermediary's fee, what are the realized net cash flows if LIBOR is 8.25 percent?

ANSWER:

| <u>FI A</u> | (in millions of dollars) | <u>FI B</u> | |
|--|--------------------------|------------------------------|------------------|
| Pays out fixed rate | (\$10.00) | Pays out LIBOR | (\$8.25) |
| Receives LIBOR from B | \$8.25 | Receives fixed rate from A | \$10.00 |
| Pays floating-rate to creditors (LIBOR+0.5%) | (\$8.75) | Pays fixed-rate to creditors | <u>(\$10.00)</u> |
| Pays intermediary fee | <u>(\$0.10)</u> | | |
| Net cash inflow | (\$10.60) | Net cash inflow | (\$8.25) |

This solution is an extension of the diagram in part (c) and the explanation at the beginning of part (d) above where LIBOR is 8.25 percent. The summary shows the effective cost rate converted to dollars for the total cash flows of each FI. However, the cash flows in a swap arrangement include only the differential cash flows between the two parties. Thus, at the end of the year, FI A would pay \$1.75m (\$10.00m - \$8.25m) to FI B and \$0.10m to the intermediary for a total cash flow on the swap arrangement of \$1.85m. FI B receives \$1.75m from FI A.

- f. If FI A buys the swap in part (e) from FI B and pays the swap intermediary's fee, what are the realized net cash flows if LIBOR is 11 percent? Be sure to net swap payments against cash market payments for both FIs.

ANSWER:

| <u>FI A</u> | (in millions of dollars) | <u>FI B</u> | |
|--|--------------------------|------------------------------|------------------|
| Pays out fixed rate | (\$10.00) | Pays out LIBOR | (\$11.00) |
| Receives LIBOR from B | \$11.00 | Receives fixed rate from A | \$10.00 |
| Pays floating-rate to creditors (LIBOR+0.5%) | (\$11.50) | Pays fixed-rate to creditors | <u>(\$10.00)</u> |
| Pays intermediary fee | <u>(\$0.10)</u> | | |
| Net cash inflow | (\$10.60) | Net cash inflow | (\$11.00) |

Even though LIBOR has increased to 11 percent, FI A's total effective cost rate has not changed. The rate remains at 10.60 percent, or a total of \$10.60 million. However, the cost rate for FI B has increased because LIBOR has increased. Thus, the actual cash flows in the swap transaction now are such that FI B pays \$1.00m (\$11m - \$10m) to FI A, and that FI A receives \$1.00m and pays out \$0.10m to the intermediary. Each FI, of course, must pay the cash market liability rates.

- g. If all barriers to entry and pricing inefficiencies between FI A's debt markets and FI B's debt markets were eliminated, how would that affect the swap transaction?

ANSWER: If relative prices are the same in the markets of both FI A and FI B, then there are no potential gains to trade and therefore no swap transaction can take place. Each FI will issue debt in their respective debt markets.

24.12. An FI has \$500 million of assets with a duration of nine years and \$450 million of liabilities with a duration of three years. The FI wants to hedge its duration gap with a swap that has fixed-rate payments with a duration of six years and floating-rate payments with a duration of two years. What is the optimal amount of the swap to effectively macrohedge against the adverse effect of a change in interest rates on the value of the FI's equity?

ANSWER: Using the formula,

$$N_s = [(D_A - kD_L)A]/(D_{\text{Fixed}} - D_{\text{Floating}}) = [(9 - 0.9 \times 3)\$500 \text{ million}]/(6 - 2) = \$787.5 \text{ million.}$$

24.15. Bank A has the following balance sheet information (in millions):

| <u>Assets</u> | | <u>Liabilities and Equity</u> | |
|-----------------------|--------------|-------------------------------|--------------|
| Rate-sensitive assets | \$50 | Rates-sensitive liabilities | \$75 |
| Fixed-rate assets | <u>150</u> | Fixed-rate liabilities | 100 |
| | | Net worth | <u>25</u> |
| Total assets | <u>\$200</u> | Total liabilities and equity | <u>\$200</u> |

Rate-sensitive assets are repriced quarterly at the 91-day Treasury bill rate plus 150 basis points. Fixed-rate assets have five years until maturity and are paying 9 percent annually. Rate-sensitive liabilities are repriced quarterly at the 91-day Treasury bill rate plus 100 basis points. Fixed-rate liabilities have two years until maturity and are paying 7 percent annually. Currently, the 91-day Treasury bill rate is 6.25 percent.

a. What is the bank's current net interest income? If Treasury bill rates increase 150 basis points, what will be the change in the bank's net interest income?

ANSWER: Interest income = $50(0.0625 + 0.015) + 150(0.09) = \17.375 million, and interest expense = $75(0.0625 + 0.01) + 100(0.07) = \12.4375 million. Thus, net interest income = \$4.9375 million.

After the interest rate increase, interest income = $50(0.0775 + 0.015) + 150(0.09) = \18.125 million, interest expense = $75(0.0775 + 0.01) + 100(0.07) = \13.5625 million, and net interest income = \$4.5625 million for a decline of \$375,000.

b. What is the bank's repricing or funding gap? Use the repricing model to calculate the change in the bank's net interest income if interest rates increase 150 basis points.

ANSWER: Funding gap = Rate sensitive assets - Rate sensitive liabilities = $50 - 75 = -\$25$ million. The repricing model states that $\Delta \text{NII} = \text{GAP}(\Delta R) = -25(0.015) = -\0.375 million. The bank is exposed to interest rate increases since interest expense increases more than interest income.

- c. *How can swaps be used as an interest rate hedge in this example?*

ANSWER: A short hedge can be used to hedge the bank's interest rate risk exposure. The short hedge can be implemented by selling futures or forward contracts, buying put options, or buying a swap of liabilities. Swapping liabilities allows the institution to make fixed-rate liability payments in exchange for a counter-party making the floating rate payments. Similarly, the FI could also swap assets. The short hedge can be accomplished by swapping out fixed-rate asset payments in exchange for floating-rate asset payments.

24.16. *Use the following information to construct a swap of asset cash flows for the bank in problem 15. The bank is a price taker in both the fixed-rate market at 9 percent and the rate-sensitive market at the T-bill rate plus 1.5 percent. A securities dealer has a large portfolio of rate sensitive assets funded with fixed-rate liabilities. The dealer is a price taker in a fixed-rate asset market paying 8.5 percent and a floating-rate asset market paying the 91-day T-bill rate plus 1.25 percent. All interest is paid annually.*

- a. *What is the interest rate risk exposure to the securities dealer?*

ANSWER: The securities dealer is exposed to interest rate declines.

- b. *How can the bank and the securities dealer use a swap to hedge their respective interest rate risk exposures?*

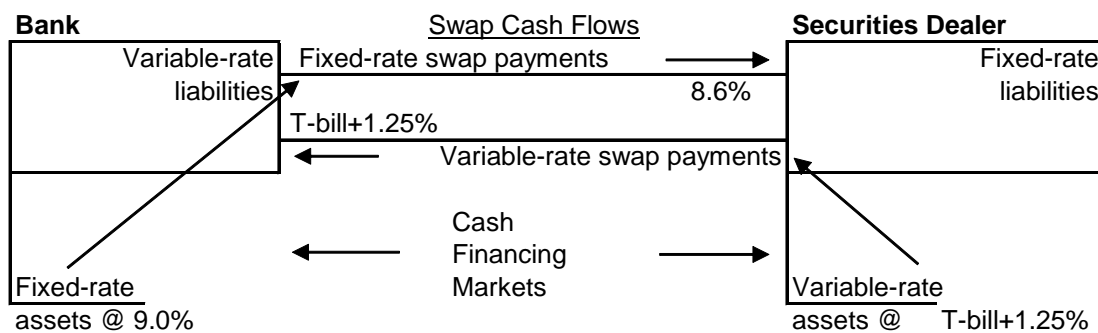
ANSWER: The two counterparties can use a swap of asset cash flows to hedge their respective interest rate risk exposures. The bank would swap out fixed-rate asset payments in exchange for floating-rate asset payments to yield positive cash flows when interest rates increase. The securities dealer would swap out floating-rate asset payments in exchange for fixed-rate asset payments to yield positive cash flows when interest rates decrease.

- c. *What are the total potential gains to the swap?*

ANSWER: The total gains to the swap trade are 25 basis points. This is because the bank earns a 25 basis point premium in the floating-rate market and a 50 basis point premium in the fixed-rate market.

- d. *Consider the following two-year swap of asset cash flows: An annual fixed-rate asset cash flow of 8.6 percent in exchange for a floating-rate asset cash flow of T-bill plus 125 basis points. The swap intermediary fee is 5 basis points. How are the swap gains apportioned between the bank and the securities dealer if they each hedge their interest rate risk exposures using this swap?*

ANSWER: See the Flow Chart below. The securities dealer gains 10 basis points because it obtains fixed-rate cash inflows at 8.6 percent instead of the 8.5 percent available in its cash market. The bank gains 10 basis points because it obtains floating rate cash inflows at T-bill + 1.60 percent instead of the T-bill + 1.50 percent available in its cash market. The remaining 5 basis points goes to the swap intermediary.



| | <u>Bank</u> | <u>Securities Dealer</u> |
|-----------------------------|---------------------|--------------------------|
| Cash market asset rate | 9.00% | T-bill+1.25% |
| Minus swap rate | -8.60% | -(T-bill+1.25%) |
| Plus swap rate | <u>T-bill+1.25%</u> | <u>8.60%</u> |
| Net financing cost rate | T-bill+1.65% | 8.60% |
| Minus swap intermediary fee | <u>-0.05%</u> | |
| | T-bill+1.60% | |
| Minus cash market rate | <u>T-bill+1.50%</u> | <u>8.50%</u> |
| Net gain from swap | 0.10% | 0.10% |

e. What are the realized cash flows if T-bill rates at the end of the first year are 7.75 percent and at the end of the second year are 5.5 percent? Assume that the notional value is \$107.14 million.

ANSWER:

At the end of the first year (in millions of dollars):

| | Bank Cash Flows | | Securities Dealer Cash Flows |
|--------------------------|------------------------|---------------------------|-------------------------------------|
| Swap cash inflows | | | |
| 107.14(0.0775 + 0.016) = | \$10.0176 | 107.14(0.086) = | \$9.214 |
| Cash market cash flows | | | |
| 107.14(0.09) = | <u>\$9.6426</u> | 107.14(0.0775 + 0.0125) = | <u>\$9.6426</u> |
| Net swap gain (loss) | \$0.375 | | (\$0.4286) |

The dealer pays the bank \$375,000 to offset the decline in net interest income when interest rates increase (see part a) and pays the swap intermediary \$53,600 (5 basis points), for a total cost of \$428,600 when interest rates increase 150 basis points.

At the end of the second year, interest rates decline to 5.5%.

| | Bank Cash Flows | | Dealer Cash Flows |
|--------------------------|------------------------|--------------------------|--------------------------|
| Swap cash inflows | | | |
| 107.14(0.0550 + 0.016) = | \$7.607 | 107.14(0.086) = | \$9.214 |
| Cash market cash flows | | | |
| 107.14(0.09) = | <u>\$9.6426</u> | 107.14(0.055 + 0.0125) = | <u>\$7.232</u> |
| Net swap gain (loss) | (\$2.036) | | \$1.982 |

The bank pays the dealer \$1.982 million and pays the swap intermediary \$53,600 (5 basis points), for a total cost of \$2.036 million when interest rates decrease 75 basis points.

f. *What are the sources of the swap gains to trade?*

ANSWER: The gains to the swap trade emanate from the pricing discrepancy in the two cash markets. That is, the bank earns a 50 basis point premium in the fixed-rate asset market, while only a 25 basis point premium in the floating rate asset market. The swap allows both the bank and the securities dealer to exploit their own comparative advantage in their respective cash market.

g. *What are the implications for the efficiency of cash markets?*

ANSWER: There must be some barrier that prevents the two firms from directly transacting in the other's cash market (or equivalently raises the costs of these cross-market transactions). This barrier may consist of regulatory restrictions or tax considerations. If, however, the barrier results from information asymmetries, these potential gains to trade can be expected to disappear as the swap market develops.

24.20. *What is a total return swap?*

ANSWER: A total return swap involves swapping an obligation to pay interest at a specified fixed or floating rate for payments representing the total return on a loan or a bond of a specific amount. The swap can be designed to cover any change in value of the principal as well as just the interest. This type of swap often is used when there is exposure to a change in the credit risk of the counterparty.

24.21. *How does a pure credit swap differ from a total return swap? How does it differ from a digital default option?*

ANSWER: The total return swap includes an element of interest rate risk, while the pure credit swap has stripped this risk from the contract. In a pure credit swap, the lender makes a fixed fee or payment premium to the counterparty in exchange for the potential coverage of any loss due to a specific borrower defaulting on a loan. The swap is not tied to interest rate changes. The pure credit swap is similar in payoff to a digital default option with the exception that the premium is paid over the life of the swap rather than at the initiation of the risk coverage as with the option.

24.22. *Why is the credit risk on a swap lower than the credit risk on a loan?*

ANSWER: The credit risk on a swap is lower than that of a loan for the following reasons:

- a) Swaps do not involve the exchange of principal payments. They only involve the swapping of interest payments, so the most a counterparty can lose is the difference in the interest payments.
- b) In most cases, payments are made through netting by novation, which nets all payments with one counterparty, further reducing the possibility of default.
- c) Swaps made by parties with poor credit ratings are usually backed by lines of credit, effectively making them collateralized loans, and further reducing their risks.